Solutions for Tutorial 8
The PID Algorithm

8.1 The Proportional-integral-derivative (PID) controller algorithm involves simple calculations. Why was this important during the development of the algorithm and for the practice of process control?

The PID controller was developed long before digital computation was available for process control; it was developed in the 1930's, while digital control began in the 1960's. Therefore, the controller calculations had to be implemented using the concepts of analog computation, in which a physical system was designed and built that followed the equations to the solved. For process control, pneumatic computers were used. Their dynamic behavior, basically described by Newton's laws, were matched to behave like the PID equation. For this to be possible, the equation was required to be simple.

However, the PID controller can give quite acceptable performance for many process applications. As a result, the PID is available in essentially every digital control system. It is the "work horse" of process control because a high percentage the valves in the process industries are regulated by the PID algorithm.

Many new and more powerful algorithms have been developed for demanding process applications. Even in these cases, the PID is typically used to provide basic control, with the advanced algorithm at a higher level in a hierarchy. We call this cascade control and will learn about it in Chapter 14.

8.2 The statement is made that the feedback controller affects stability and damping. Demonstrate that this statement is correct for a proportional-only controller. Use the three-tank mixer model from Example 7.2.

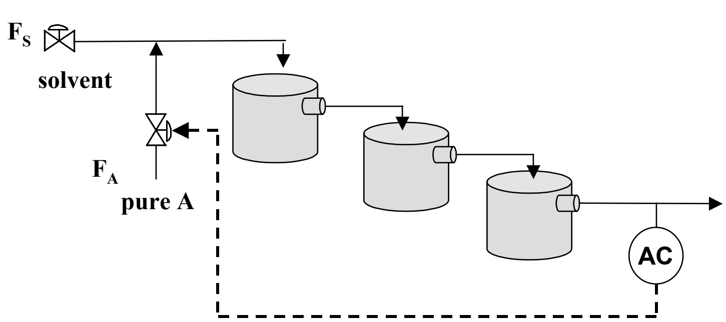


Figure 8.2

We know that the transfer function relating an input-output pair for a feedback control system is given in the following equation.

$$\frac{CV(s)}{SP(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$

We also know that we can determine the stability and damping of the system by evaluating the roots of the characteristic equation, i.e., the denominator of the transfer function. We will use the following models (individual transfer functions) for the elements in the characteristic equation.

Proportional controller: $G_C(s) = K_C$

Three-tank process: $G_P(s)G_v(s)G_S(s) = \dfrac{0.039}{(5s+1)^3}$

Substituting and rearranging, the characteristic equation is determined.

$$\begin{aligned}
&1 + G_p(s)G_v(s)G_c(s)G_S(s) \\
&1 + \frac{K_C K_P}{(1+\tau s)^3} = 0 \\
&(1+5s)^3 + K_C(0.039) = 0 \\
&125s^3 + 75s^2 + 15s + [1 + 0.039K_c] = 0
\end{aligned}$$

Clearly, the controller (Kc) affects the equation! The roots of the equation for various values of the controller gain are given below.

Kc =0	Kc = 50	Kc = 100	Kc = 150
-0.2000	-0.4499	-0.5148	-0.5604
-0.2000 + 0.0000i	-0.0751 + 0.2164i	-0.0426 + 0.2726i	-0.0198 + 0.3121i
-0.2000 - 0.0000i	-0.0751 - 0.2164i	-0.0426 - 0.2726i	-0.0198 - 0.3121i

Columns 5 through 6

Kc = 200	Kc = 250
-0.5966	-0.6273
-0.0017 + 0.3435i	0.0136 + 0.3700i
-0.0017 - 0.3435i	0.0136 - 0.3700i

Unstable!

We observe that the roots become complex at Kc = 50. This indicates some oscillation in the dynamic behavior. Also, at Kc = 250, two of the roots have positive real parts, which indicate unstable behavior.

8.3 **Proportional Mode**:

a. What are the units of Kc? What is the sign for stabilizing negative feedback?

a. The definition of the controller is

$$G_c(s) = K_c = \frac{MV(s)}{CV(s)}$$

Therefore, the units of the controller gain are (MV units)/(CV units). We note that these are the inverse of the units for the process gain, Kp, although Kc≠1/Kp.

We look at the controller equation to determine the sign.

$$E(t) = SP(t) - CV(t)$$

$$MV(t) = K_c \left[E(t) + \frac{1}{T_I} \int_0^{\infty} E(t')dt' - T_d \frac{d\,CV}{dt} \right] + I$$

Let's do a thought experiment, in which we will increase the set point by +1.0. Since the error is defined as (SP-CV), the error will increase, i.e., its change will be positive. Also, we assume that the process gain is positive, Kp > 0. Also, to increase the CV, we know that the controller must increase the MV. As a result, the controller gain (Kc) must be positive. We leave as additional exercises other combinations of positive and negative set point changes and process gains.

After considering all combinations, we conclude that the product of the process gain times the controller gain must be positive to give negative feedback control, KpKc > 0.

8.3 **Integral Mode**:

a. Determine the final value of the error from set point for a PI controller applied to a first order process in response to a first-order disturbance. The disturbance is an **impulse** in the feed concentration of A in the solvent stream.

b. Determine the final value of the error from set point for a PI controller applied to a first order process in response to a first-order disturbance. The disturbance is a **step** in the feed concentration of A in the solvent stream.

c. Determine the final value of the error from set point for a PI controller applied to a first order process in response to a first-order disturbance. The disturbance is a **ramp** in the feed concentration of A in the solvent stream.

The deviation for the error from set point is exactly the deviation of the controlled variable (CV) from its initial value. The closed-loop transfer function for this system is given below.

$$\frac{CV(s)}{D(s)} = \frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$

We will determine the final value by substituting the specific disturbance input function and applying the Final Value Theorem.

a. The disturbance is an impulse; its Laplace Transform is L(impulse) = C, with C being a constant.

$$CV(s) = D(s)\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$
$$= C\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$

$$\lim_{s\to 0} sCV(s) = \lim_{s\to 0} s\, C\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)} = 0$$

We see that the PI controller provides zero-steady-state offset for an impulse disturbance. In fact, a proportional-only controller would achieve the same desirable behavior; the verification is left as an exercise for you to complete.

b. The disturbance is an step; its Laplace Transform is L(step) = C/s, with C being a constant.

$$CV(s) = D(s)\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$
$$= \frac{C}{s}\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$

$$\lim_{s\to 0} sCV(s) = \lim_{s\to 0} s\frac{C}{\mathbf{s}}\frac{K_d /(\tau_d s+1)}{1+K_C\left(1+ \frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)} = 0$$

We see that the PI controller provides zero-steady-state offset for a step disturbance. Would we obtain the same desirable result for a Proportional-only controller?

c. The disturbance is an ramp; its Laplace Transform is L(ramp) = C/s^2, with C being a constant.

$$CV(s) = D(s)\frac{K_d/(\tau_d s+1)}{1+K_C\left(1+\frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$

$$= \frac{C}{s^2}\frac{K_d/(\tau_d s+1)}{1+K_C\left(1+\frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)}$$

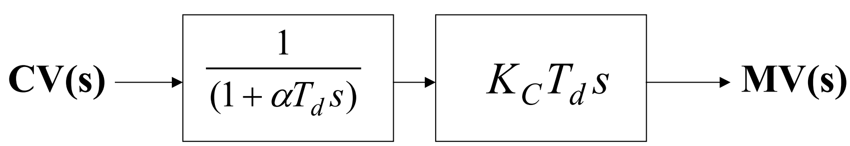
$$\lim_{s\to 0} sCV(s) = \lim_{s\to 0} s\frac{C}{\mathbf{s^2}}\frac{K_d/(\tau_d s+1)}{1+K_C\left(1+\frac{1}{T_I s}\right)\left(\frac{K_P}{(\tau_P s+1)}\right)} = \frac{K_d}{K_C K_P / T_I} \neq 0$$

We see that the PI controller does not provide zero-steady-state offset for a ramp disturbance. Would the result change if we added a derivative mode to the controller?

8.4 **Derivative Mode**: The derivative mode is described as a exact derivative. Rather than exact derivative, it is often implemented using the equation below, which is the Laplace Transform for the function. Suggest a reason for using the modified derivative mode calculation in the following equation.

$$\text{Derivative mode:} \quad G_C(s) = \frac{MV(s)}{CV(s)} = K_C\frac{T_d s}{1+\alpha T_d s}$$

The transfer function can be separated into two series calculations that help to understand the overall behavior of the modified derivative mode.



First order filter

Exact derivative

The first term is a filter that reduces the "noise" in the signal. The parameter alpha (α) is small, usually about 0.10, so that the filter does not unduly slow the response of the derivative. The second term is the exact derivative which acts on the signal after filtering.

The goal is to have an effective derivative mode without amplifying the high frequency noise in the measured variable. The modified calculation is effective when the noise is if much higher frequency than the dynamics of the process variable, i.e., the critical frequency of the feedback system (see Chapter 10 for the evaluation of the critical frequency).

8.5 A PID controller must be initialized every time it is "turned on" (or placed in automatic) by the plant personnel. Some data is given for the situation when the controller is placed in automatic; the controller equation is also given. Perform the initialization calculation.

$$E(t) = SP(t) - CV(t)$$
$$MV(t) = K_c \left[E(t) + \frac{1}{T_I} \int_0^{\infty} E(t')dt' - T_d \frac{d\,CV}{dt} \right] + I$$

Data:

Data:	Set point	= 100 °C
	Measured controlled variable	= 98 °C
	Derivative of the controlled variable	≈ 0
	Signal to control valve	= 63.7 % open
	Controller Gain, Kc	= 2.30 %/°C
	Controller integral time	= 4.50 minutes
	Controller derivative time	= 0.67 minutes

The initialization calculation determines the bias constant (I), so that the valve does not "jump" when the controlled is turned on. We call this bumpless transfer.

The derivative is zero based on the data, and the integral mode is zero, because the value of time is zero when the controller starts its calculation.

Now, we calculate the bias (I) so that the first calculation does not change the signal to the valve.

$$E(t) = SP(t) - CV(t) = 2$$
$$MV(t) = K_c[E(t)] + I =$$
$$= 2.3(2) + I = 63.7$$

$$I = 59.1 \quad \%open$$

The signal to the valve, MV(t), will not change at the instant that the controlled is placed in operation. The bias is never changed after the initialization calculation, so that the controller can change the valve and control the CV!

