Centralized Multivariable Control

CHAPTER 23

23.1 ▣ INTRODUCTION

The first three chapters in this section on multivariable control retained the proportional-integral-derivative (PID) control algorithm. This approach is generally preferred for its simplicity when it provides good performance, which is often the case. However, some especially challenging process control objectives are difficult or impossible to achieve using multiloop PID control. In this chapter, one centralized method for controlling multiple input-output processes is introduced. The term *centralized* denotes a control algorithm that uses all (process input and output) measurements simultaneously to determine the values of all manipulated variables. In contrast, multiloop control, also called *decentralized* control, involves many algorithms, with each using only one process output variable to determine the value of one manipulated variable. Further discussions on the need for centralized control are presented in Cutler and Perry (1983) and Prett and Garcia (1988).

In addition to all measurements, centralized controllers use a dynamic model of the process in the control calculation. The most common approach to using a model explicitly in the control calculation is the model predictive control structure described in Chapter 19. Since the discussions in this chapter are based on an understanding of the model predictive structure, the reader is advised to review Chapter 19 thoroughly before proceeding with this chapter.

This chapter begins with a straightforward extension of the model predictive controller to a multivariable system. This extension demonstrates the limitations in applying the analytical model inverse, which was easily determined for single-variable systems, to the multivariable case. Then, one approach to determining a controller design using numerical methods to obtain good dynamic performance is presented, first for single-variable and subsequently for multivariable systems. In this chapter, the digital algorithm is presented, because of the clarity and ease of implementation of this form. The presentation of the new control algorithm is concluded with discussions on implementation guidelines and extensions.

23.2 ▣ MULTIVARIABLE MODEL PREDICTIVE CONTROL

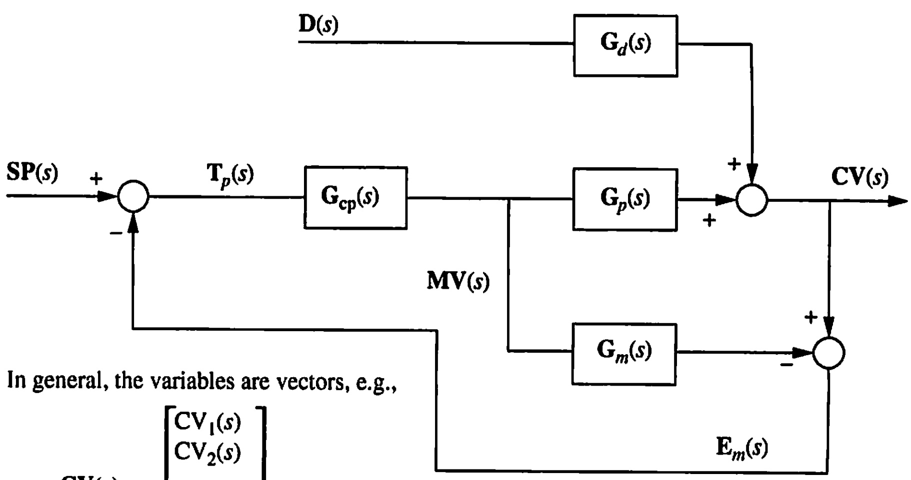
Model predictive control was introduced in Chapter 19, where some important properties were demonstrated for single-loop systems. The same principles can be applied to a multivariable system. For example, the following properties can be shown to hold for the general (open-loop stable) system in Figure 23.1.

1. The controlled variables will return to their set points for steplike inputs if

$$\mathbf{G}_{cp}(0) = [\mathbf{G}_m(0)]^{-1} \tag{23.1}$$

   Thus, the steady-state gain matrix of the controller must be the same as the inverse of the steady-state process model. Again, this can be achieved easily, because the engineer selects both of these elements in the control system. Note that the model does not have to match the plant exactly, although large model mismatch can degrade performance and lead to instability.
2. Perfect control (i.e., zero deviation from set point) is achieved when

$$\mathbf{G}_{cp}(s) = [\mathbf{G}_m(s)]^{-1} \tag{23.2}$$



In general, the variables are vectors, e.g.,

$$\mathbf{CV}(s) = \begin{bmatrix} CV_1(s) \\ CV_2(s) \\ \vdots \\ CV_{NC}(s) \end{bmatrix}$$

The transfer functions are matrices of appropriate dimensions, e.g.,

$$\mathbf{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

with $\mathbf{G}_{ij}(s)$ relating input j to output i.

FIGURE 23.1

Model predictive control structure.

Even if possible, this might involve excessive variability in the manipulated variable and thus not be desirable in practice.

3. If the model (and process) contains noninvertible elements, an *approximation* to equation (23.2) can be used to determine the controller, as follows:

$$G_{cp}(s) = \left[G_m^-(s)\right]^{-1} \tag{23.3}$$

$$G_m(s) = G_m^+(s)\, G_m^-(s) \tag{23.4}$$

with $G_m^+(s)$ The "noninvertible" factor has an inverse that is not causal or is unstable. The inverse of this term includes predictions, $e^{\theta s}$, and unstable poles, $1/(1+\tau s)$, $\tau < 0$, appearing in $[G_m(s)]^{-1}$. The steady-state gain of this factor must be the identity matrix.

$G_m^-(s)$ The "invertible" factor has an inverse that is causal and stable, leading to a realizable, stable controller. The steady-state gain of this factor is the gain matrix of the process model, K_m.

For single-variable systems, the design of the controller $G_{cp}(s)$ was relatively straightforward. However, the application of this analytical approach to multivariable systems encounters a significant barrier, as demonstrated in the following example.

EXAMPLE 23.1.

A multivariable predictive controller is to be applied to the binary distillation tower considered throughout the book. The product compositions are to be controlled by adjusting the reflux and reboiler; thus, the energy balance regulatory control strategy provides the base control on which the composition control will be implemented. This approach, which shows the multivariable controller as an upper-level component in a cascade design, is given in Figure 23.2.

The model for the process is given in equation (21.1) and is repeated here:

$$\begin{bmatrix} X_D \\ X_B \end{bmatrix} = \begin{bmatrix} \dfrac{0.0747e^{-3s}}{12s+1} & \dfrac{-0.0667e^{-2s}}{15s+1} \\ \dfrac{0.1173e^{-3.3s}}{11.75s+1} & \dfrac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R \\ F_V \end{bmatrix} + \begin{bmatrix} \dfrac{0.70e^{-5s}}{14.4s+1} \\ \dfrac{1.3e^{-3s}}{12s+1} \end{bmatrix} X_F \tag{23.5}$$

This two-variable system would be represented in the general symbols of Figure 23.1 as

$$\begin{bmatrix} CV_1(s) \\ CV_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} MV_1(s) \\ MV_2(s) \end{bmatrix} + \begin{bmatrix} G_{d1}(s) \\ G_{d2}(s) \end{bmatrix} D(s) \tag{23.6}$$

By applying equation (23.2) the predictive controller is evaluated by determining the inverse of the feedback model.

$$[G_m(s)]^{-1} = \frac{1}{\dfrac{0.00782e^{-5.3s}}{(15s+1)(11.75s+1)} - \dfrac{0.00936e^{-5s}}{(12s+1)(10.2s+1)}} \begin{bmatrix} \dfrac{-0.1253e^{-2s}}{10.2s+1} & \dfrac{0.0667e^{-2s}}{15s+1} \\ \dfrac{-0.1173e^{-3.3s}}{11.75s+1} & \dfrac{0.0747e^{-3s}}{12s+1} \end{bmatrix} \tag{23.7}$$

The model in equation (23.5) cannot be factored uniquely into an invertible part. Also, the control performance of a control system that would satisfy equations

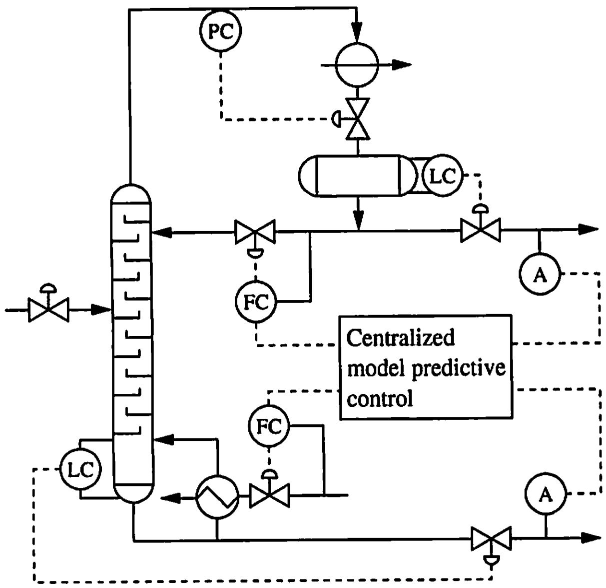


FIGURE 23.2

Centralized multivariable distillation control.

Relative volatility	2.4
Number of trays	17
Feed tray	9
Analyzer dead times	2 min
Feed light key	X_F = 0.50
Distillate light key	X_D = 0.98 mole fraction
Bottoms light key	X_B = 0.02 mole fraction
Feed flow	F_F = 10.0 kgmole/min
Reflux flow	F_R = 8.53 kgmole/min
Distillate flow	F_D = 5.0 kgmole/min
Reboiler flow	F_V = 13.53 kgmole/min
Tray holdup	H = 1.0 kgmole
Holdup in drums	HD = 10.0 kg mole

(23.1) and (23.3) is not easily related to the analytical method of obtaining the invertible factor $\mathbf{G}_m^-(s)$.

> Thus, the analytical algorithm design method in equations (23.1) through (23.4) will not be used for multivariable systems in this chapter, although the model predictive structure will be retained.

The distillation example will be reconsidered after an alternative controller algorithm has been developed.

23.3 ▣ AN ALTERNATIVE DYNAMIC MODELLING APPROACH

The previous section demonstrated that a new approach to designing the model predictive algorithm is needed. Fortunately, several approaches have been developed, and one of these will be presented in the next section. However, the new method requires dynamic models in a format different from the standard transfer functions used to this point. The requisite modelling is described in this section using the symbols X for input and Y for output. This convention is used because these models can represent the input-output behavior for various variable combinations; for example, X could represent a disturbance or a manipulated variable.

Throughout the book, transfer function models have been determined from fundamental modelling and empirical identification. These transfer functions are very useful in representing the dynamic input-output behavior of linear (or linearized) elements in a control system. They are parsimonious, in that the entire dynamic response can be represented by a small number of parameters. Also, their analytical structure enables the engineer to perform many transformations and calculations easily. However, alternative model structures are possible. For example, a dynamic model can be represented by the two forms in Table 23.1. This is the model of a single-tank mixing process with transportation delay used for PID tuning studies in Section 9.3, and it will be used in examples later in this chapter.

The example transfer function considered here is first-order with dead time, but more complex equations are common and can be modelled using this approach. An alternative model form is the step response, which is a set of discrete values representing the output response to a unit step input; these values are often referred to as the *step weights*. The transfer function gives a continuous model of the process, whereas the step response gives no information at times between the sampled points and has the same values as the continuous model at the sample points. The step response can be developed from the transfer function by solving for the output response of the continuous system to a unit (+1) step input at sample number 0. For the example first-order-with-dead-time system, the discrete form is

$$Y(s) = \frac{K_p e^{-\theta s}}{(\tau s + 1)} X(s) = \frac{K_p e^{-\theta s}}{s(\tau s + 1)} \quad [\text{note } X(s) = \Delta X/s = 1/s] \tag{23.8}$$

$$Y'(t) = (1.0) K_p (1 - e^{-(t-\theta)}) \quad (t \geq \theta) \tag{23.9}$$

This continuous model can be evaluated at sample points by setting time equal to

TABLE 23.1

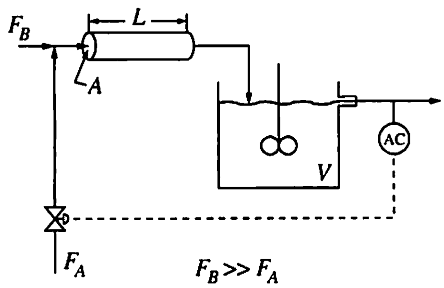
Transfer function with its step response model

Transfer function

$Y(s)/X(s) = K_p e^{-\theta s}/(\tau s + 1)$
$= 1.0 e^{-5s}/(5s + 1)$

Step response

Sample k	Time t	$X'(t)$	$Y'(t) = a_k$
0	0	1	0.
1	2.5	1	0.
2	5	1	0.
3	7.5	1	0.394
4	10	1	0.632
5	12.5	1	0.777
6	15	1	0.865
7	17.5	1	0.918
8	20	1	0.950
9	22.5	1	0.970
10	25	1	0.982
11	27.5	1	0.989



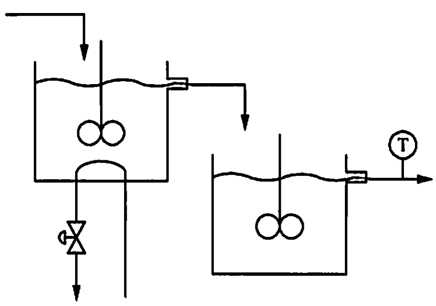
multiples of the sample period, Δt. In the following equations the subscripts m emphasize that the transfer function parameters refer to the model, which is only an approximate representation of the true plant.

Time	Sample no.	Input X'	Output Y'
0	0	1	0
Δt	1	1	0
$2\Delta t$	2	1	0
	{continues until the dead time}		
θ_m	$\theta_m/\Delta t$	1	0
$\theta_m + \Delta t$	$(\theta_m + \Delta t)/\Delta t$	1	$K_m(1 - e^{-\Delta t/\tau_m})$
$\theta_m + 2\Delta t$	$(\theta_m + 2\Delta t)/\Delta t$	1	$K_m(1 - e^{-2\Delta t/\tau_m})$
$\vdots$	$\vdots$	1	$\vdots$

The reader can verify that this method was used to develop the step weights from the transfer function in Table 23.1 by calculating the step response from an initial steady state of $Y' = 0$. The step response model can be used to calculate the value of the output Y' at any sample period k in response to a step of any size ΔX using the equation

$$Y_k' = a_k \Delta X_0 \tag{23.10}$$

Recall that the transfer function used in developing the step response can be derived from fundamental models using methods from Chapters 3 through 5, or it can be developed from empirical data using methods from Chapter 6.



EXAMPLE 23.2.

Determine two models for the data in Figure 23.3: a transfer function model and a discrete step model.

The continuous transfer functions can be determined using the methods described in Chapter 6. This data was used in Examples 6.2 and 6.6, where it was concluded that a first-order-with-dead-time structure was adequate. For example, the parameters determined in Example 6.6 using the statistical parameter method are $K_m = 2.56°\text{C}/\%$ open, $\theta_m = 3.66$ min, and $\tau_m = 5.2$ min.

The discrete step response can also be determined from the data. One approach would be to use the measured values of the output variables as the step response; this approach would use the data indicated by the circles in Figure 23.3. While this represents the process behavior exactly for *this* experiment, the data includes noise, which would not be repeatable and should not be used for designing or tuning controllers. A better method for determining the step response would characterize the repeatable process response and ignore the higher-frequency noise. There are many methods for evaluating a step model from noisy data; one good method uses conventional modelling methods (for example, those in Chapter 6) to fit a transfer function model and subsequently evaluate the step response using the transfer function. This approach is demonstrated in Figure 23.3, where the dashed line is the continuous output from the transfer function model and the crosses are the step response from the estimated *model*, not the raw data. This

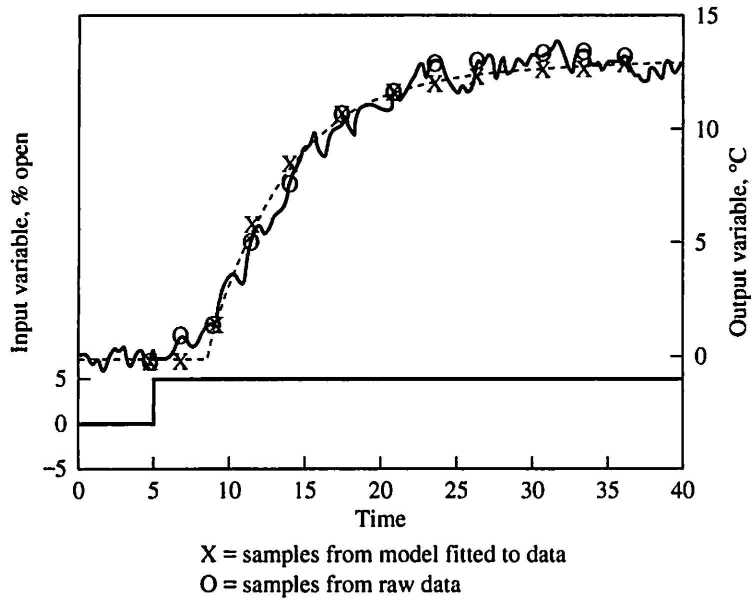


FIGURE 23.3

Process reaction data with continuous and discrete models.

modelling approach captures the dominant dynamic behavior while eliminating the effects of most of the noise. Further discussions of determining representative step response models from empirical data are given in MacGregor et al. (1991), Cutler and Yocum (1991), and Ricker (1988).

The step response model can be used to predict the dynamics of a system for any input function of time. This is achieved by sampling the input function and recognizing that it can be approximated by step changes at each sample point. The effect on the output of each input step is represented by the step response in equation (23.10). The overall effect of all of the input steps is the sum of each individual effect, assuming that the system is *linear.* This modelling method introduces potential errors, because the input may not be a perfect staircase function; however, the errors will be small if the sample period is short compared with the rate of change of the input and output variables. Assuming that the plant begins at a steady-state condition (Y_0), the step weights can be used to predict the output from the input values at the sample points as follows:

$$\begin{aligned} Y_1 &= Y_0 + a_1 \Delta X_0 \\ Y_2 &= Y_0 + a_2 \Delta X_0 + a_1 \Delta X_1 \\ Y_3 &= Y_0 + a_3 \Delta X_0 + a_2 \Delta X_1 + a_1 \Delta X_2 \end{aligned} \tag{23.11}$$

and so forth. This model can be expressed as an equation for any number of sample periods k for a single-input–single-output system as follows:

$$Y_{k+1} = Y_0 + \sum_{j=1}^{k+1} a_j \Delta X_{k-j+1} \tag{23.12}$$

Applying the model in equation (23.12) for a long time ($k \to$ large) would result in a sum over a very large number of samples, since every change in the past influences the current value of the output variable. We anticipate that such a large summation would cause difficulties for the controller calculation. However, the input changes have a constant effect as the time from the input step becomes large; that is, after the transient settles to the constant effect for a past ΔX. Thus, the model in equation (23.12) can be rewritten to give the following equation.

$$Y_{k+1} = \underbrace{Y_0}_{\text{Initial condition}} + \underbrace{\sum_{j=\mathrm{LL}+1}^{k+1} a_j \Delta X_{k-j+1}}_{\text{Reached steady state}} + \underbrace{\sum_{j=1}^{\mathrm{LL}} a_j \Delta X_{k-j+1}}_{\text{Transient response}} \tag{23.13}$$

The last term on the right-hand side includes those past inputs whose effects have not yet reached their steady-state values. Thus, the number of samples multiplied by the period should be the settling time of the process; for example, $\mathrm{LL}\Delta t$ is approximately equal to the dead time plus four time constants for a first-order-with-dead-time process. The second term on the right-hand side involves the inputs whose effects have (essentially) reached their steady state, so that $a_k \approx K_p$ for $k > \mathrm{LL}$. It is not necessary to sum all of the values in the second term at each time step, because the summation only changes by one value each sample period: by 1 past ΔX. Thus, this can be calculated recursively using a new intermediate variable Y^* to include the initial value of Y and the effects of all ΔX values whose effects have reached steady state. (Recall that a recursive calculation uses only the past result and the new input to calculate the new result.)

$$\begin{aligned} Y_k^* &= Y_{k-1}^* + a_{\mathrm{LL}+1} \Delta X_{k-\mathrm{LL}} \\ Y_{k+1} &= Y_k^* + \sum_{j=1}^{\mathrm{LL}} a_j \Delta X_{k-j+1} \end{aligned} \tag{23.14}$$

The approximation of the step response with its steady-state (or final) value introduces another potential error, which can be made small by proper choice of the number of steps (LL) to include in the summation in equation (23.13). Now the large sum of the steady-state effects has been eliminated by the recursive form of the model.

The step response model in equation (23.14) does not require all past inputs to be stored and the large summation to be calculated each execution: the information about initial condition and inputs whose effects have reached steady state are accumulated in the Y^* term.

The modelling approach described in this section can be applied to most single input–single output responses; it cannot be applied to *unstable processes*, for which Y^* (all past effects that have reached steady state) does not exist. Because only discrete samples of the response are used, the step response model is not as complete a representation as a continuous transfer function model. However, the discrete step response model facilitates the design of centralized feedback controllers, as explained in the next section.

23.4 ▣ THE SINGLE-VARIABLE DYNAMIC MATRIX CONTROL (DMC) ALGORITHM

Several approaches can be used to develop a practical multivariable centralized controller. The method presented here is the dynamic matrix controller, which was developed by Cutler (Cutler and Ramaker, 1979), was extended to include additional features (Prett and Gillette, 1979; Garcia and Morshedi, 1986), and has been applied successfully to complex processes (e.g., Kelly et al., 1988; Van Hoof et al., 1989). The dynamic matrix control algorithm can be implemented within the model predictive control structure, and the algorithm can be designed without determining the analytical inverse of the process model, so the extension to multivariable systems is straightforward. The DMC algorithm will be introduced here for the single-variable case and then will be extended to multivariable. This explanation will proceed in three steps, each introducing a key aspect of the overall algorithm.

Basic DMC Algorithm (without Feedback)

The algorithm will be introduced by considering the situation encountered every time a model predictive feedback controller is executed. The dynamic response of a feedback control system is shown in Figure 23.4. The manipulated variable has been adjusted in the past, and the controlled variable has been influenced by these adjustments, as well as by disturbances. The prediction of the controlled variable, calculated using equations (23.14) and past values of the manipulated variable, is also shown in the figure. The task of the control algorithm is to determine *future* adjustments to the manipulated variable that will result in the predicted controlled variable returning quickly to the set point.

To determine the best controller moves, a measure of control performance must be selected. Here, the integral of the error squared, or the sum of the error squared at sample points, will be taken; we recognize that this measure is not complete, and we will modify it later to consider robustness and the behavior of the manipulated variable. The error—deviation between set point and controlled variable—can be measured at the current time, but we know that it will change in the future because of recent adjustments to the manipulated variable. The behavior of the controlled variable *without adjustments in the future* should be used to determine the future error, which should be reduced by future adjustments. Thus, the DMC controller uses a dynamic model of the process to calculate the future behavior of the controlled variable that would occur without future control adjustments.

$$\mathrm{CV}_i^f = \mathrm{CV}_K^* + \sum_{j=1}^{\mathrm{LL}} a_{j+i} \Delta \mathrm{MV}_{K-j} \qquad \text{Note: Without feedback} \qquad (23.15)$$

with CV_i^f = predicted (deviation) value of the controlled variable in the future as influenced by past changes in the manipulated variable

CV_K^* = predicted value of the controlled variable at the current time based on all past inputs up to $K - \mathrm{LL}$

i = sample periods in the future ($i = 1$ to NN)

The difference between the predicted values of the controlled variable and the set point are used to calculate the objective, the sum of the errors squared, which is to

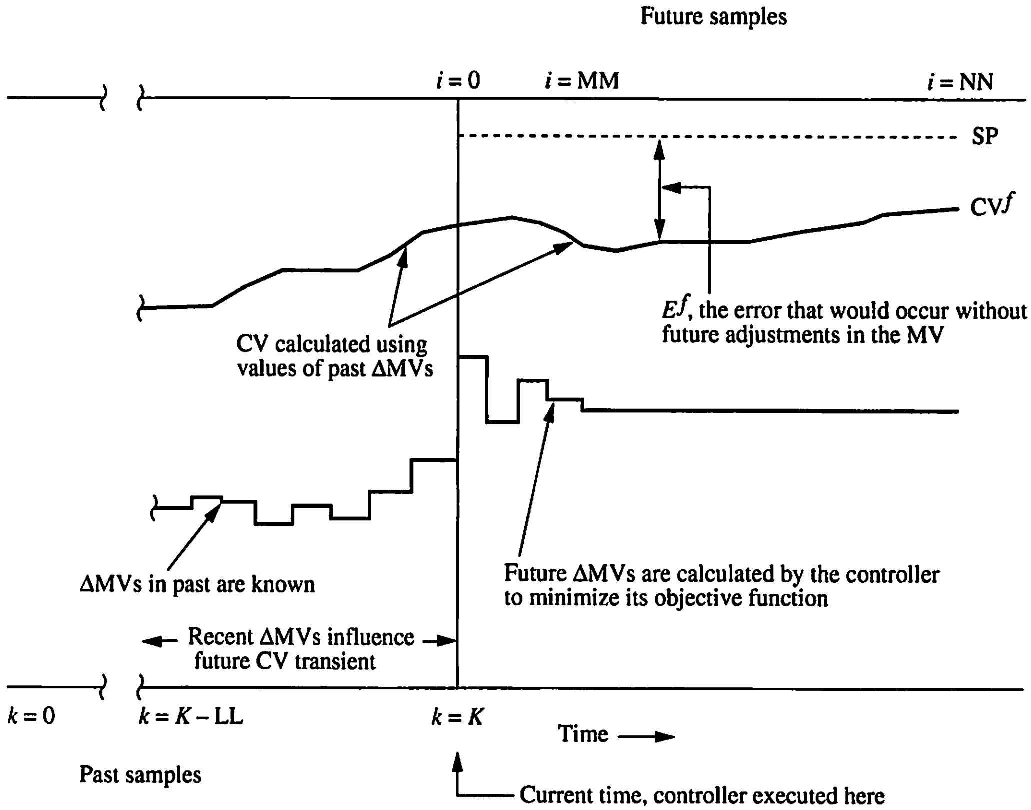


FIGURE 23.4

Dynamic response of variables for DMC control.

be minimized.

$$\text{OBJ}_{\text{DMC}} = \sum_{i=1}^{\text{NN}} \left[\text{SP}_i - (\text{CV}_i^f + \text{CV}_i^c)\right]^2 = \sum_{i=1}^{\text{NN}} \left[E_i^f - \text{CV}_i^c\right]^2 \tag{23.16}$$

where
- SP_i = set point at each sample i in the future
- CV_i^f = defined in equation (23.15) and cannot be influenced by the controller
- CV_i^c = effect of future adjustments on the controlled variable at each sample i
- E_i^f = $(\text{SP}_i - \text{CV}_i^f)$, deviation from set point that would occur if no future control adjustment were made
- NN = future time over which the control performance is evaluated, termed the *output horizon*

In equation (23.16), the set point can remain constant at its current value in the future, but if it will vary in the future in a manner known when the controller is executed, a variable set point can be accommodated. Also, the future effects of *past* adjustments, CV_i^f, are calculated using equation (23.15). Thus, only the terms CV_i^c are influenced by the *future* adjustments determined by the controller algorithm. Finally, the output horizon (NN) should be long enough for the controlled variable to approach steady state under closed-loop control.

Now, the challenge is to determine the future adjustments in the manipulated variables to minimize the objective. This is an optimization problem that could be solved by many methods, including searching over a large grid of possible values of the manipulated adjustments, but that would involve wasteful, excessive calculations. An efficient controller calculation method can be developed using the modelling approach introduced in the previous section. The step response model can be used to calculate the effects of *future* moves, by summing their effects.

$$CV^c_{i+1} = \sum_{j=1}^{i+1} a_j \Delta MV^c_{i-j+1} \tag{23.17}$$

where CV^c = effects of future adjustments in the manipulated variable on the controlled variable

ΔMV^c = future adjustments calculated by the controller

This model can be slightly rearranged to ease the optimization calculation. The same result can be obtained with the summation over all inputs at each sample i of the horizon by ensuring that the effects are zero for all adjustments occurring after the sample at which the controlled variable is evaluated (i). This model can be expressed in matrix format as follows, using the step weights a_j that can be nonzero (where $i \geq j$) and 0.0 for the elements that must be zero (where $i < j$).

$$\begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{NN} & a_{NN-1} & a_{NN-2} & \cdots & a_{NN-MM+1} \end{bmatrix} \begin{bmatrix} \Delta MV^c_0 \\ \Delta MV^c_1 \\ \Delta MV^c_2 \\ \vdots \\ \Delta MV^c_{MM-1} \end{bmatrix} = \begin{bmatrix} CV^c_1 \\ CV^c_2 \\ CV^c_3 \\ \vdots \\ CV^c_{NN} \end{bmatrix} \tag{23.18}$$

In this formulation, the adjustments in the manipulated variable could be allowed for all samples in the output horizon; however, experience indicates that this can lead to overly aggressive control action and oscillatory dynamic responses. Therefore, fewer manipulated-variable adjustments are allowed, and the number of adjustments is given by the *input horizon* MM, which must be less than the output horizon. Equations (23.17) and (23.18) are equivalent, and either one may be used to evaluate the effects of future adjustments on the control objective. Perhaps equation (23.18) provides a clearer picture of the calculation. The coefficient matrix in equation (23.18) is often designated by the symbol **A** and is referred to as the *dynamic matrix.* With this notation, equation (23.18) can be rewritten as

$$\mathbf{A}[\Delta MV^c] = [CV^c] \tag{23.19}$$

The goal of perfect controlled-variable performance would be to have zero error for all samples in the future, which would be achieved if

$$\mathbf{E}^f = [CV^c] \quad \text{perfect control of } \mathbf{CV} \tag{23.20}$$

However, this performance cannot be achieved in general, because of dead times, constraints on the manipulated variables, and right-half-plane zeros (see Sections 13.5 and 19.2). Another way of stating this conclusion is that an exact plant (model) inverse cannot be achieved because of limitations in the physical process. Therefore, the best control involves the manipulated-variable adjustments that minimize the sum of the error squared in equation (23.16), which in general is not zero. The solution to this problem is the least squares solution, which can be considered an approximate plant (model) inverse that has desirable properties for control performance. The solution to the optimization problem in equation (23.16) for the model in equation (23.18) is the well-known linear least squares result

$$\mathbf{K}_{\mathrm{DMC}} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}} \tag{23.21}$$

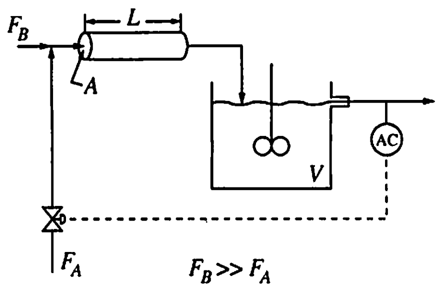
The dynamic matrix controller $\mathbf{K}_{\mathrm{DMC}}$ can be used to calculate the future adjustments at each controller execution by

$$\mathbf{K}_{\mathrm{DMC}}\mathbf{E}^{f} = [\Delta \mathrm{MV}^{c}] \tag{23.22}$$

This equation shows that the model of the process in the feedback path, **A**, and the future errors are used to calculate the manipulated-variable adjustments. The calculated adjustment for the current time period, $\Delta \mathrm{MV}_0^c$, would be implemented after the controller calculation. The later adjustments would not be implemented, because they would be recalculated during later controller calculations.

EXAMPLE 23.3.

The process model in Table 23.1 describes the mixing process with dead time. Feedback control using the proportional-integral-derivative (PID) algorithm has been evaluated for this process in Section 9.3. Assume that the process is initially at steady state, and its set point is changed by a 1% step. Design the DMC controller matrix and evaluate the closed-loop dynamic response, assuming that the model is perfect.



The following parameters must be chosen before the DMC design calculation can be performed.

Δt = sample period
LL = number of sample periods required for the process model to reach steady state
NN = controlled-variable (output) horizon
MM = manipulated-variable (input) horizon

In this example, the analyzer update occurs only once every 2.5 min; thus, the controller execution is set by this limitation. The product $(\Delta t)(\mathrm{LL})$ should be equal to or greater than the settling time of the open-loop process, and the product $(\Delta t)(\mathrm{NN})$ should be equal to or greater than the settling time of the closed-loop process. The manipulated-variable horizon is usually selected to be greater than 1, to allow some manipulated-variable overshoot if desired, and to settle well before the end of the controlled-variable horizon; thus, $1 \le \mathrm{MM} \le \mathrm{NN}$. The values of the parameters for this example are summarized in Table 23.2. The horizons are somewhat shorter than usually used in practice, to enable the key matrices to be reported conveniently.

Since the system is initially at steady state, so that all past adjustments are zero, the future errors are equal to the current error. Some of the key values in the calculation of the future moves follow.

$$\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0.394 & 0.632 & 0.777 & 0.865 & 0.918 & 0.950 & 0.970 & 0.982 & 0.989 \\ 0 & 0 & 0 & 0.394 & 0.632 & 0.777 & 0.865 & 0.918 & 0.950 & 0.970 & 0.982 \\ 0 & 0 & 0 & 0 & 0.394 & 0.632 & 0.777 & 0.865 & 0.918 & 0.950 & 0.970 \\ 0 & 0 & 0 & 0 & 0 & 0.394 & 0.632 & 0.777 & 0.865 & 0.918 & 0.950 \end{bmatrix}$$

$$\mathbf{K}_{DMC} = \begin{bmatrix} 0 & 0 & 2.54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.08 & 2.54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.53 & -4.08 & 1.86 & 0.95 & 0.40 & 0.065 & -0.14 & -0.26 & -0.34 \\ 0 & 0 & 0 & 1.54 & -2.04 & -0.93 & -0.26 & 0.14 & 0.39 & 0.54 & 0.62 \end{bmatrix}$$

$$(\mathbf{E}^f)^T = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$$

These values can be used to calculate the future values of the manipulated-variable changes using equation (23.22). The changes can be summed to obtain the manipulated-variable values at each time in the future.

$$[\Delta MV^c] = \begin{bmatrix} 2.54 \\ -1.54 \\ 0.00 \\ 0.00 \end{bmatrix} \qquad MV^c = \begin{bmatrix} 2.54 \\ 1.00 \\ 1.00 \\ 1.00 \end{bmatrix}$$

Because the controller model is assumed perfect in this example, feedback does not change the results in later controller executions. The responses of the manipulated and controlled variables are given in Figure 23.5. The controlled variable cannot respond until after the process dead time, and for this system it can be changed to the set point in one sample period after the dead time. To achieve this performance, the manipulated variable must experience a rapid change of large magnitude, which may not be acceptable. However, the controller objectives, as stated to this point, have been achieved.

TABLE 23.2

Summary of single-variable DMC simulation cases

	Algorithm parameters						Controlled-variable performance		MV performance
Case	**Δt**	**MM**	**NN**	**ww**	**qq**	**Controller model, difference from plant model****	**IAE**	**ISE**	**$\sum(\Delta MV)^2$**
Example 23.3 Figure 23.5	2.5	4	11	1	0	Same as process	6.0	5.6	8.8
Example 23.4 Figure 23.7	2.5	4	11	1	0	$K_m = 0.65$	12.9	7.7	29.5
Example 23.5 Figure 23.8*a*	2.5	4	11	1	0.2	$K_m = 0.65$	11.5	7.3	2.9
Example 23.5 Figure 23.8*b*	2.5	4	11	1	0.2	$K_m = 0.65$	8.9	4.2	0.8

*The process is represented by the model in Table 23.1.

**The model used in performing all model-based calculations for DMC.

LL = large number, e.g., 5(NN).

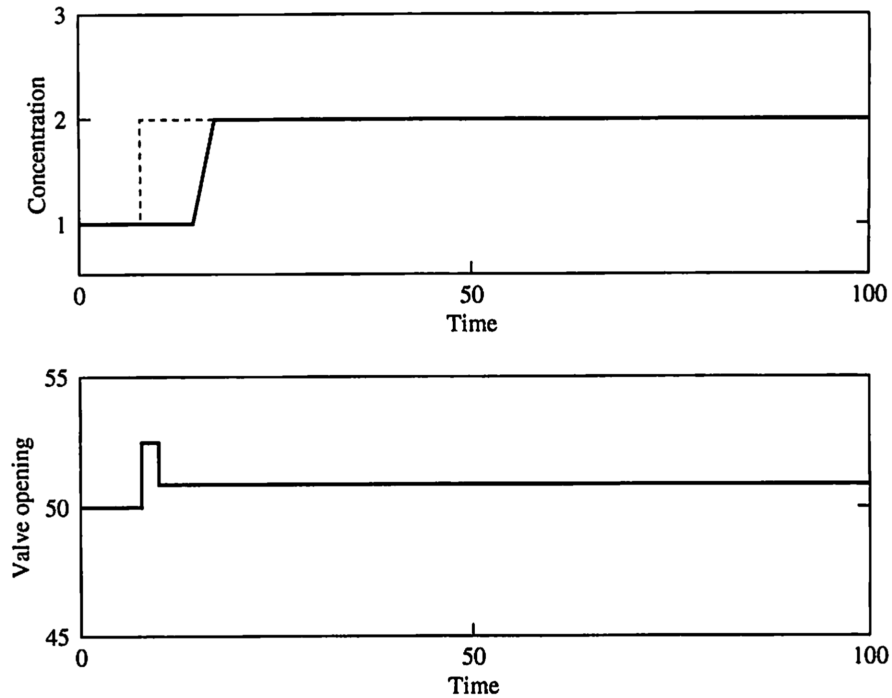


FIGURE 23.5

Dynamic response from Example 23.3 for the case with no model error.

Adding Feedback to the DMC Controller

To achieve acceptable feedback performance, the DMC controller must use the measured value of the controlled variable. The method for including the feedback is the same as employed in Chapter 19: the measured value is compared with a predicted value, and the difference, the feedback signal E_m, is added to the predicted value used by the controller. This scheme is shown in Figure 23.1; note that adding the feedback to the predicted controlled variable has the same effect on the sum of error squared as subtracting it from the set point, as seen by considering equation (23.16). This feedback approach is equivalent to adjusting a bias in the predictive model *without changing* the step weights a_j; thus, the feedback dynamics used by the controller to relate adjustments in the manipulated variable to the controlled variable are not influenced by the feedback. The result of the feedback, shown in Figure 23.6, is similar to that in the model predictive controllers in Chapter 19: zero steady-state offset for steplike disturbances, but no adaptation of dynamics for nonlinearities.

The model used to calculate the effects of future changes in the manipulated variables is similar to equation (23.15). However, the prediction of the future behavior without control is modified to combine the model with the feedback measurement signal as follows.

$$CV_i^f = CV_K^* + (E_m)_K + \sum_{j=1}^{LL} (a_{j+i})\Delta MV_{K-j} \tag{23.23}$$

The feedback signal is the difference between the measured and predicted values,

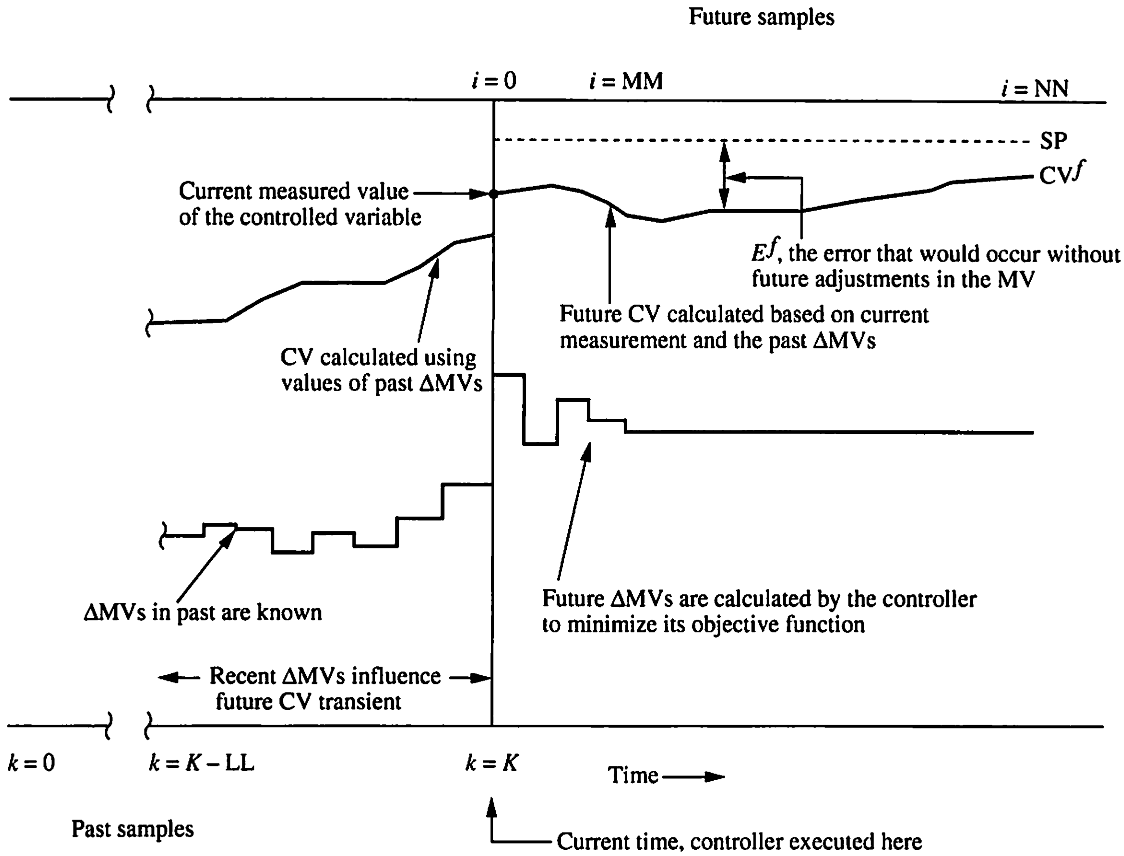


FIGURE 23.6

Dynamic response of variables with feedback.

which is assumed to remain unchanged in the future:

$$(E_m)_K = (\mathrm{CV}_{\mathrm{meas}})_K - \mathrm{CV}_K \tag{23.24}$$

Substituting equation (23.24) into equation (23.23) yields the model for the effects of future changes in the manipulated variables.

$$\mathrm{CV}_i^f = \mathrm{CV}_K^* + (\mathrm{CV}_{\mathrm{meas}})_K - \mathrm{CV}_K + \sum_{j=1}^{\mathrm{LL}} a_{j+i}\Delta \mathrm{MV}_{K-j} \tag{23.25}$$

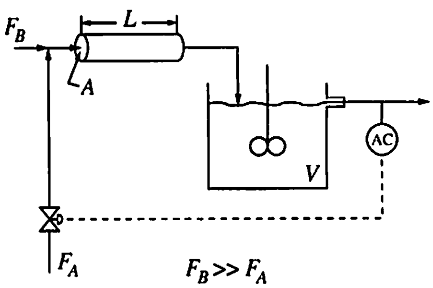
Thus, the feedback method is equivalent to setting the model prediction at the current time to the current measured value of the controlled variable.

The DMC controller $\mathbf{K}_{\mathrm{DMC}}$ can be designed with the same calculations, equation (23.21). Again only the manipulated-variable adjustment at the current sample period, $\Delta\mathrm{MV}_0$, is implemented. The entire controller calculation is repeated at the next sample period, because a new measured value of the controlled variable is available.

Some insight into the model predictive structure is gained by considering the meaning of the feedback signal when the controller model is perfect. In this situation, the effects of the manipulated variable on the true plant and the model are identical and cancel when E_m is calculated. Thus, the feedback signal is equal to the effect of the *disturbance* on the controlled variable. Since the same value of the feedback signal E_m is used to calculate all future values of the controlled variable without future adjustments, CV_i^f for all $i = 1$ to NN, the tacit assumption has been made that the disturbance will be the same in the future as it is currently. This is often a reasonable assumption when we have no special information about the disturbance.

EXAMPLE 23.4.

The results in Example 23.3 were for the case when the controller model exactly represented the true process. In this example, the model differs from the plant; the model gain is 0.65%/% open, while the process gain remains 1.0%/% open. Determine the closed-loop performance for this system.



The step response for the model can be derived using the method in Table 23.1 with $K_m = 0.65$ (not 1.0), and this model can be used to derive the DMC controller from equation (23.21). The controller can then be employed with feedback, and the resulting dynamic response is shown in Figure 23.7 and summarized in Table 23.2. The model error led to considerable oscillation in this example, with increased ISE of the controlled variable and excessive manipulated-variable variation. However, the controlled variable ultimately returned to the set point, which was the goal of the feedback. Thus, feedback has improved the performance of the closed-loop system, but its dynamic behavior is not yet acceptable.

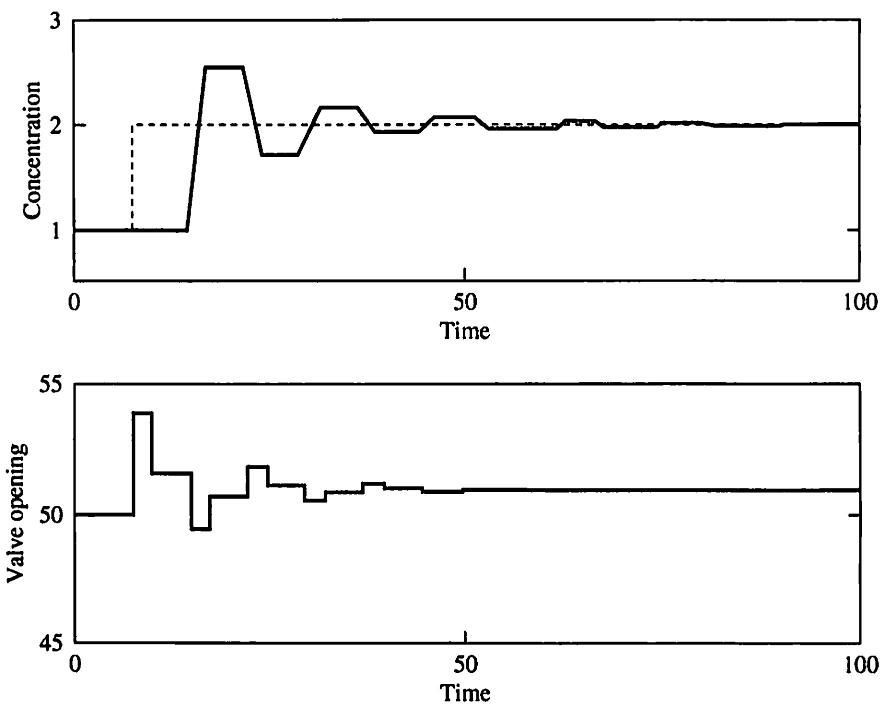


FIGURE 23.7

Dynamic response for Example 23.4 for the case with model error but no move suppression.

Adding Tuning for Manipulated-Variable Behavior and Robustness

As with all controllers, adjustable parameters are needed to match the closed-loop performance to the particular needs (manipulated-variable variability) and circumstances (model mismatch) encountered in each application. In the DMC controller, the principal manner for addressing these needs is to expand the objective used in defining the control algorithm. This is done by adding a term that penalizes changes in the manipulated variable at each execution.

$$
\begin{aligned}
\mathrm{OBJ_{DMC}} &= \sum_{i=1}^{\mathrm{NN}} \left\{ \mathrm{ww} \left[\mathrm{SP}_i - (\mathrm{CV}_i^f + \mathrm{CV}_i^c) \right]^2 \right\} + \sum_{i=1}^{\mathrm{MM}} [\mathrm{qq}(\Delta \mathrm{MV}_i)^2] \\
&= \sum_{i=1}^{\mathrm{NN}} \left[\mathrm{ww} \left(E_i^f - \mathrm{CV}_i^c \right)^2 \right] + \sum_{i=1}^{\mathrm{MM}} [\mathrm{qq}(\Delta \mathrm{MV}_i)^2]
\end{aligned}
\tag{23.26}
$$

where ww = (≥ 0) adjustable parameter weighting the controlled-variable deviations from set point, the ISE

qq = (≥ 0) adjustable parameter weighting the adjustments of the manipulated variable. This parameter is termed the *move suppression factor.*

The *relative* values of the two tuning parameters ww and qq determine how much importance is placed on the controlled variable ISE and on the variability of the manipulated variable; the original definition of the controller in equation (23.16) can be thought of as equation (23.26) with ww = 1 and qq = 0. Naturally, some variability in the manipulated variable must be allowed to enable the control system to respond to disturbances and set point changes. However, the controller with qq = 0 can be very aggressive, as seen in Figure 23.7. Also, because of model mismatch, the controller with qq = 0 can lead to an unstable closed-loop system, and increasing the value of qq (more correctly, qq/ww) increases the range of model mismatch for which stable closed-loop performance is achieved. Finally, equation (23.26) contains no term for deviations of the manipulated variable from a target value, since the manipulated variable must be free to respond to disturbances of various magnitudes and directions; thus, the penalty is on the *adjustment* or change at each sample.

Again, the control algorithm determines the values of the future manipulated-variable changes that minimize the objective function. The result is

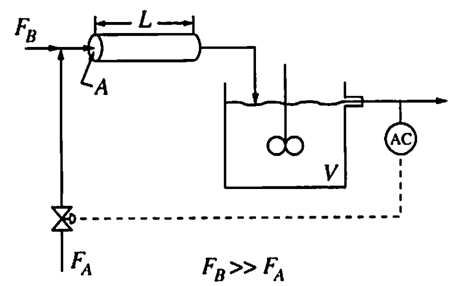
$$
\mathbf{K}_{\mathrm{DMC}} = (\mathbf{A}^{\mathrm{T}}[\mathrm{WW}]\mathbf{A} + [\mathrm{QQ}])^{-1}\mathbf{A}^{\mathrm{T}}[\mathrm{WW}] \tag{23.27}
$$

where [WW] = diagonal matrix = $\mathrm{ww}\mathbf{I}_{\mathrm{NN}}$

[QQ] = diagonal matrix = $\mathrm{qq}\mathbf{I}_{\mathrm{MM}}$

$\mathbf{I}_R$ = identity matrix of size $R \times R$

Again, only the current manipulated-variable adjustment is implemented at each controller execution. This is the form of the DMC control algorithm used in industrial practice.



EXAMPLE 23.5.

Evaluate the control performance with model mismatch in Example 23.4, using the DMC algorithm in equation (23.27) with adjustable tuning.

The matrix algebra in equation (23.27) is slightly more complex, but the required model information (i.e., step weights) is the same. In this example, the number of parameters for the engineer to select is increased with the addition of ww and qq. For the single-variable DMC, ww can be set to 1.0 without loss of generality, which is not true for the extension to multivariable. The value for qq is selected to be 0.20 for this example, and the choice of this value is discussed in Section 23.6. The resulting transient response is shown in Figure 23.8*a*, and parameters and performance values are summarized in Table 23.2. The performance with qq = 0.2 is much more acceptable, with lower ISE of the controlled variable and about one-tenth the variability of the manipulated variable ($\sum \Delta MV^2$). An additional transient response has been evaluated for this system with the same feedback model mismatch and controller tuning parameters; this is a response to a unit step disturbance with a model $G_d(s) = 1/(5s+1)$. The response in Figure 23.8*b* shows that DMC provides acceptable transient behavior and zero steady-state offset for this disturbance.

The addition of the variability of the manipulated variable to the controller objective with the associated tuning factor qq provides the engineer with the flexibility to tune the controller for a wide range of objectives and model mismatch.

23.5 ◻ MULTIVARIABLE DYNAMIC MATRIX CONTROL

It would be possible to employ the single-loop DMC as a replacement for the PID controller and to implement multiloop control with DMC using the approaches presented in Chapters 20 through 22. However, this approach would not realize the great power of dynamic matrix control (or other similar centralized multivariable algorithms). Here, the goal is to achieve *centralized* multivariable control, in which the algorithm uses information from all controlled variables to calculate all manipulated-variable adjustments simultaneously each execution. Fortunately, the nature of the DMC algorithm makes its extension to multivariable control straightforward. In addition, the calculations performed at each controller execution remain relatively simple.

Again, the basis for the algorithm is the step response model. In the multivariable situation, one model exists for each input-output combination, and the form of each single input-output model remains as described in Section 23.3. The objective for the controller becomes

$$\mathrm{OBJ_{DMC}} = \sum_{\mathrm{nc}=1}^{\mathrm{NC}} \mathrm{ww_{nc}} \sum_{i=1}^{\mathrm{NN}} \left[E^f_{\mathrm{nc},i} - \mathrm{CV}^c_{\mathrm{nc},i}\right]^2 + \sum_{\mathrm{nm}=1}^{\mathrm{NM}} \mathrm{qq_{nm}} \sum_{i=1}^{\mathrm{MM}} (\Delta \mathrm{MV_{nm,\mathit{i}}})^2 \tag{23.28}$$

where
- NC = number of controlled variables; nc is the counter for the controlled variables (1 to NC)
- NM = number of manipulated variables; nm is the counter for the manipulated variables (1 to NM)
- $\mathrm{ww_{nc}}$ = adjustable parameter weighting the nc'th controlled variable's deviation from set point
- $\mathrm{qq_{nm}}$ = adjustable parameter weighting the adjustments of the nm'th manipulated variable

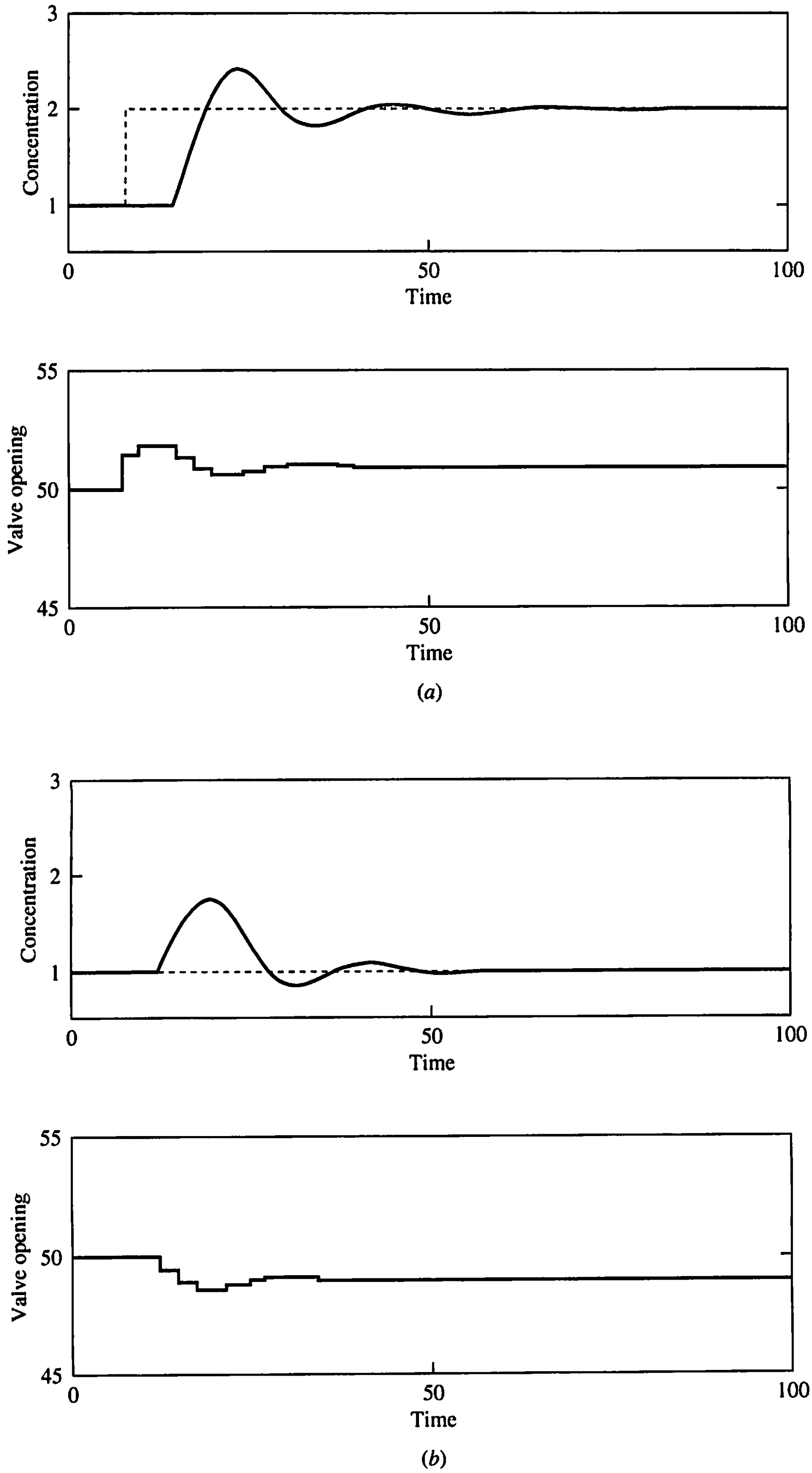


FIGURE 23.8

Dynamic response for Example 23.5 for the case with model error and move suppression: (*a*) set point change; (*b*) disturbance.

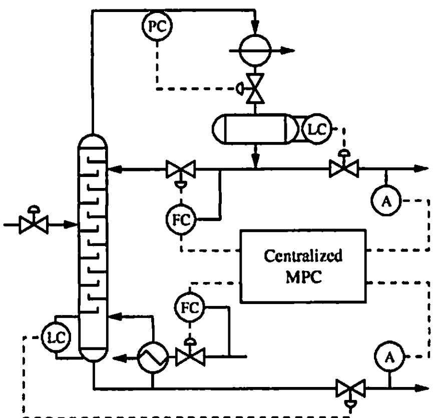
For multivariable control, a separate value for ww_{nc} is allocated to each controlled variable. The ratio of these values represents the relative importance of deviations from set point of the controlled variables, which can be used to tune the controller for different performance objectives. Also, each manipulated variable has an associated qq_{nm}, which gives the penalty for adjustments. A multivariable controller has many parameters that must be tuned in conjunction to obtain the desired performance.

The control algorithm that achieves this objective is

$$K_{DMC} = (A^T[WW]A + [QQ])^{-1}A^T[WW] \tag{23.29}$$

This is the same form as the result for single-loop control. However, the matrices in equation (23.29) are composed of individual blocks, with each block consisting of a single-variable matrix or zeros. For the two-variable control problem,

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad [WW] = \begin{bmatrix} [WW]_1 & 0 \\ 0 & [WW]_2 \end{bmatrix} \quad [QQ] = \begin{bmatrix} [QQ]_1 & 0 \\ 0 & [QQ]_2 \end{bmatrix} \tag{23.30}$$



where $A_{nc,nm}$ = dynamic matrix shown in equation (23.18) for the controlled variable nc and the manipulated variable nm
$[WW]_{nc}$ = diagonal matrix = $ww_{nc}I_{NN}$ for nc = 1 to NC
$[QQ]_{nm}$ = diagonal matrix = $qq_{nm}I_{MM}$ for nm = 1 to NM
$\mathbf{0}$ = square matrix containing zeros

With this result, the errors of all controlled variables are considered simultaneously in determining the adjustments to all manipulated variables. The methods of modelling and feedback conform to the model predictive control structure, with all models being multiple-input–multiple-output and the controlled and manipulated variables being vectors of values for each variable. As before, only the current manipulated-variable adjustments are implemented at each controller execution.

EXAMPLE 23.6.

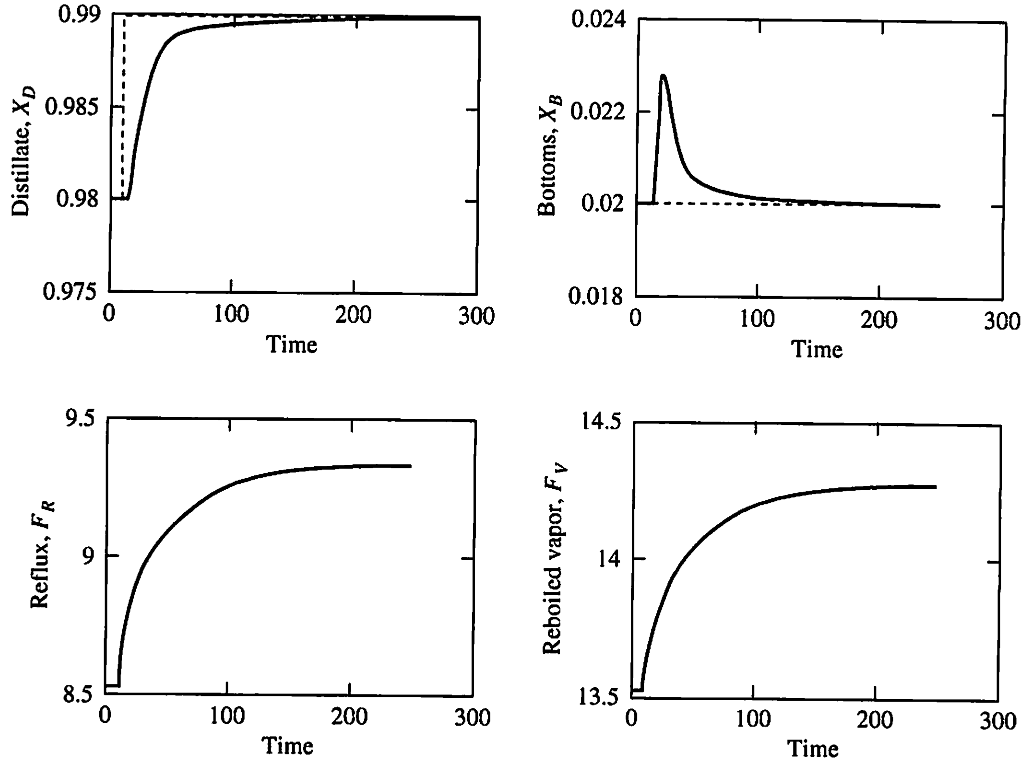
Apply DMC control to the distillation tower considered in Example 23.1 and modelled in equation (23.5).

The first step is to develop the step response models using the method in Table 23.1 or, equivalently, equation (23.10) for each of the $A_{nc,nm}$ matrices. In performing this modelling, the sample period and horizon lengths must be decided. Finally, the tuning parameters are selected. With this information, the DMC controller K_{DMC} can be calculated using equation (23.29). The controller design parameters used in this example are as follows:

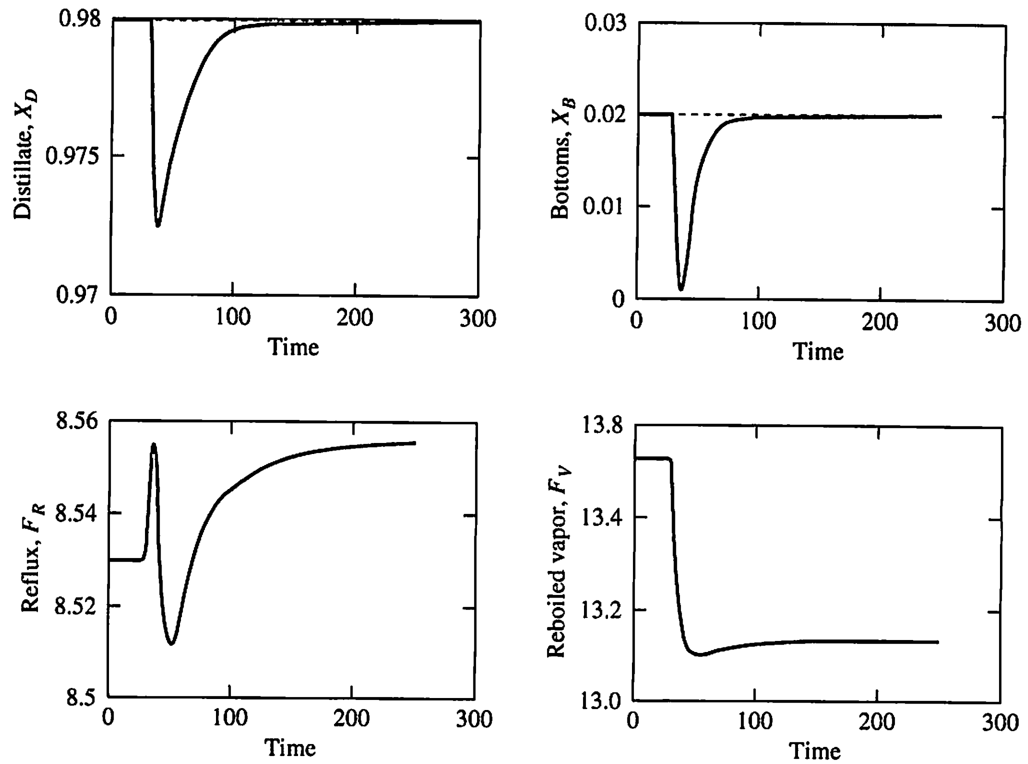
$$\Delta t = 1 \quad MM = 5 \quad NN = 20 \quad LL = 100 \quad ww_1 = ww_2 = 1 \quad qq_1 = qq_2 = 0.02$$

The transient response to a set point change in the distillate purity, with the bottoms set point unchanged, is given in Figure 23.9*a*. Also, the transient response to a −4% step change to the light key in the feed is given in Figure 23.9*b*. In both plots the plant is represented by the linear model in equation (23.5), so that these responses are for no model mismatch, although the tuning has been selected to give a reasonably moderate feedback response.

The performance measures for these dynamic responses are given in Table 23.3. These results can be compared with the performance achieved with two PI controllers, although no attempt was made to provide the best tuning to either control system. (Note that the set point change in Table 21.2 was half the magnitude



(*a*)



(*b*)

FIGURE 23.9

Responses for Example 23.6: (*a*) to set point change; (*b*) to disturbance.

TABLE 23.3

Summary of performance for Example 23.6

Case	IAE_{XD}	ISE_{XD}	IAE_{XB}	ISE_{XB}	$\sum(\Delta F_R)^2$	$\sum(\Delta F_V)^2$
Figure 23.9*a*	0.225	0.00122	0.073	0.00010	0.0141	0.0097
Figure 23.9*b*	0.207	0.00093	0.33	0.00413	0.00029	0.0147

of that in Table 23.3.) In these examples, the DMC controller provided about the same performance for the disturbance and better performance for the set point change.

As demonstrated in this section, the multivariable dynamic matrix controller is a straightforward extension of the single-variable controller. The controller algorithm can be calculated for any (stable) process model, without regard for dead times or numerator dynamics. The dynamic responses in the example show that good performance can be achieved without excessive adjustments of the manipulated variables.

23.6 ▣ IMPLEMENTATION ISSUES IN DYNAMIC MATRIX CONTROL

While the design and implementation of centralized feedback control have been shown to be possible, a large number of design and implementation decisions must be made to achieve good performance. Some of the most important are discussed briefly in this section.

Real-Time Calculations

The distinction is important between the design calculations, which are performed once offline, and the control calculations, which are performed every control execution. Basically, the design calculation is given in equation (23.29). This calculation involves the inverse of a square matrix with dimensions (MM)(MM). This inverse could be computationally intensive, but it is calculated only during offline design. In contrast, the controller calculation requires the following limited calculations every execution:

1. Calculate the feedback signal $\mathbf{E}_m$, which requires advancing the prediction of the model $\mathbf{G}_m$ in the block diagram and equation (23.14) by one time step.
2. Calculate the future error that would occur without future adjustments, $E_i^f = ([SP]_i - [CV^f]_i)$ for $i = 1, NN$. This requires the model for $[CV^f]_i$ in equation (23.23) to be calculated for NN time steps.
3. Calculate the current adjustment to the manipulated variable. The basis for this calculation is equation (23.22), which will give the adjustments for the entire input horizon—more information than needed, because only the current

change in manipulated variable is required. For example, the single-variable DMC needs only ΔMV_0, which is the sum of the element products of the top row of $\mathbf{K}_{DMC}$ and the future error E^f. This vector-vector product requires fewer calculations.

Tuning

The dynamic matrix controller has a large number of adjustable parameters, all of which influence the control performance. In addition, the best value of some parameters depend on the values of others. The following comments should help in selecting good initial values.

Δt — Factors in selecting the execution period are the same as discussed in Chapter 11 on PID control. This should be a small fraction of the closed-loop dynamics [e.g., $\Delta t < 0.1(\theta + \tau)$].

NN — The output horizon should be long enough for the closed-loop system to approach its steady state in the time Δt(NN). Typical values for NN range from 20 to 50.

MM — The input horizon is selected to be shorter than the output horizon. Typically, MM is about one-fourth to one-third of the output horizon.

ww_{nc} — The weighting for each controlled variable represents the relative importance of each deviation from its set point. Increasing this number tends to reduce the deviation of this controlled variable, but the deviation of other controlled variables will increase. The engineer must recognize that the controller objective is calculated in engineering units, so that the weighting must reconcile the comparison of various variables, such as temperature and mole percent.

qq_{nm} — The weighting for each manipulated variable represents the relative importance of the *adjustments* to each manipulated variable. Increasing this number will tend to slow the feedback adjustments, which would degrade the controlled-variable performance; however, increasing qq_{nm} also improves the robustness of the closed-loop system to model mismatch. Also, increasing qq_{nm} reduces the variability of the manipulated variable, which may be required in some circumstances. As a result, the parameter is often referred to as the *move suppression factor.*

Good values for ww_{nc} and qq_{nm} depend on their relative magnitudes, such as ww_1/qq_2 and ww_1/ww_2. Thus, strong interactions exist among the effects of the many tuning parameters on the control performance, and often some simulation studies are required to determine good tuning.

The presentation in this chapter has assumed that the weighting matrices [WW] and [QQ] are diagonal. This assumption is valid when the desired behavior of one controlled variable does not depend on the behavior of other controlled variables. That condition might not be the case for some processes. For example, a high temperature and high reactant concentration might be a particularly bad condition; in such a case, a penalty could be introduced in the appropriate off-diagonal elements in [WW] for the deviations of both.

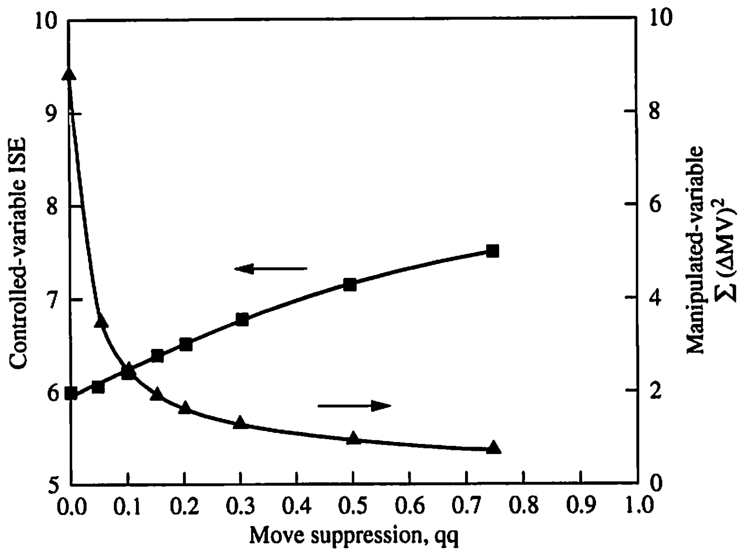
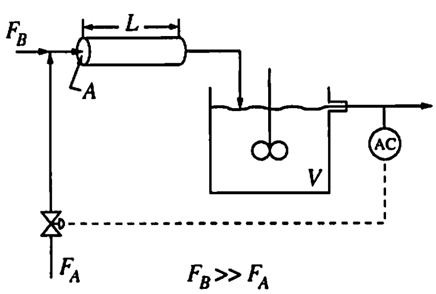


FIGURE 23.10

Effect of controller tuning on controlled and manipulated variables (ww = 1).



EXAMPLE 23.7.

Study the effects of tuning on the single-variable DMC controller in Examples 23.3 through 23.5. For this study, assume that no model mismatch exists and that the input forcing is a unit step set point change.

The common manner for presenting such a tuning study is to plot the performances of the manipulated and controlled variables against the tuning parameter, which for the single-loop case is qq/ww. This plot is given in Figure 23.10, with results that are typical of many systems. As the move suppression is increased from zero, the first effects are a rapid drop in the variability in the manipulated variable, with a small increase in the ISE of the controlled variable. After some value of qq, the effects on both variables are moderate. Often, the value of qq where the variability in the manipulated variable stops decreasing rapidly gives an acceptable initial tuning, with reasonable robustness to typical model mismatch and moderate variability in the manipulated variable. This study provided the basis for the value of qq, 0.2, used in Example 23.5.

Filtering

High-frequency noise in the controlled-variable measurement can be filtered for the reasons discussed in Section 12.3. The measurement can be filtered before calculating the feedback signal $\mathbf{E}_m$.

Cascade Implementation

Centralized multivariable controllers can output directly to final elements, but a more common design is to output to a single-loop system. As an example, the distillation control in Figure 23.2 and studied in Example 23.6 outputs to the set points of two flow controllers. The design of these lower-level loops follows the principles of single-loop enhancements (Part IV) and loop pairing (Chapters 20 through 22) already presented.

23.7 ☐ EXTENSIONS TO BASIC DYNAMIC MATRIX CONTROL

The method presented in detail in this chapter represents only the most basic form of the dynamic matrix controller. Many extensions are possible, and some are essential for success in challenging applications. A few of the more important extensions are introduced briefly in this section.

Nonsquare Systems

Many control systems have an unequal number of controlled and manipulated variables. Methods for addressing these situations using single-loop (decentralized) control were presented in Chapter 22 on variable-structure control. The DMC controller can accommodate this situation, because no assumption has been made in developing the design for $\mathbf{K}_{\mathrm{DMC}}$ in equation (23.29) regarding the number of process variables. If more controlled than manipulated variables exist, not all controlled variables can be maintained at their set points (at the steady state), and the DMC controller will minimize the objective in equation (23.26). When a steady state is achieved after a disturbance, the deviations of each controlled variable from its set point depend on the weights, ww_i. If more manipulated than controlled variables exist, all controlled variables can be maintained at their set points (in the steady state), and the manipulated variables can be adjusted to achieve additional benefits, such as low energy consumption. Methods are described in Cutler and Ramaker (1979) and Morshedi et al. (1985).

Feedforward

The centralized control method in this chapter addressed feedback control, but it can be extended to include feedforward compensation. If a measured disturbance satisfies the feedforward design criteria in Table 15.1, it can be included by modelling its effect on the future controlled variable without feedback adjustment. Thus, the effect of the measured disturbance is simply another process input in calculating the values of $[\mathrm{CV}^f]_i$ that are used in calculating $\mathbf{E}^f$ and the controller calculation in equation (23.22). Both the controller design equation for $\mathbf{K}_{\mathrm{DMC}}$ and the calculation at each controller execution, $[\Delta\mathrm{MV}] = [\mathbf{K}_{\mathrm{DMC}}][\mathbf{E}^f]$, remain unchanged.

Constraints on Variables: Quadratic Dynamic Matrix Control (QDMC)

Often, the behavior of the control system in a real plant is limited by constraints. These constraints can be limitations to manipulated variables; e.g., a valve cannot exceed 100% open or the reflux should not decrease below a minimum for proper tray contacting. In addition, constraints can be imposed on the dependent, controlled variables; for example, the temperature should not go above 350 K. The design of the DMC controller in equation (23.29) was based on a least squares method that relies on the controlled and manipulated variables having continuous derivatives, which is not valid when constraints are encountered. Fortunately, the DMC approach can be extended to designs that minimize the same objective while observing constraints by using a different optimization method. One common approach uses a solution method termed quadratic programming; thus, the controller is termed *quadratic dynamic matrix control* (QDMC). A slight disadvantage of including constraints is an increase in the calculations that must be performed with each controller execution. However, with powerful digital computers, this has not proved to be a barrier to practical application.

The measure of control performance used in QDMC is the same as the DMC controller, so that $\text{OBJ}_{\text{QDMC}} = \text{OBJ}_{\text{DMC}}$, which is given in equation (23.28). A summary of the mathematical problem solved at each controller execution is given in the following:

$$\min_{\Delta \text{MV}^c} \mathbf{OBJ}_{\text{QDMC}} \tag{23.31}$$

As in DMC, the dynamics responses between manipulated and controlled variables are represented by step-weight models.

$$[\text{CV}^c] = \text{A}[\Delta \text{MV}^c] \tag{23.19}$$

The value of the controlled variable in the future output horizon, CV_i, is

$$\text{CV}_i = \underset{\substack{\uparrow \\ \text{Calculated} \\ \text{from the future} \\ \text{adjustments,} \\ \Delta \text{MV}_i^c}}{\text{CV}_i^c} + \underset{\substack{\uparrow \\ \text{Calculated} \\ \text{from the past} \\ \text{MV and } E_m}}{\text{CV}_i^f}$$

subject to the following constraints that are imposed on every variable at every time step in the future horizon (i).

Rate of change of the MV:	$\Delta \text{MV}_{\text{min}} \le \Delta \text{MV}_i^c \le \Delta \text{MV}_{\text{max}}$
Full value of MV:	$\text{MV}_{\text{min}} \le \text{MV}_i \le \text{MV}_{\text{max}}$
Value of the CV:	$\text{CV}_{\text{min}} \le \text{CV}_i \le \text{CV}_{\text{max}}$

Some problems occur when the controlled variables are subject to strict limits as shown above; e.g., it may not be possible to achieve the controlled variable performance ($\text{CV}_{\text{min}} \le \text{CV}_i \le \text{CV}_{\text{max}}$) when the manipulated variables are also restricted. Therefore, the bounds on the controlled variables are usually implemented as penalty functions that force the solution to obey the constraints only when possible. Further details on the QDMC algorithm are provided by Garcia and Morshedi (1986), Morshedi et al. (1985), and Ricker (1985). In addition, Qin and Badgewell (1997) provide an overview of centralized model predictive control, along with a summary of similar algorithms used commonly in industry. Now, we will consider two examples of QDMC.

Typically, centralized model-predictive control is applied to plants in which substantial interaction occurs among important variables. An example of a situation with strong interaction is given in Figure 23.11, which shows a hydrocracking chemical reactor in a petroleum refinery. The process involves a series of packed bed, adiabatic reactors in which highly exothermic chemical reactions occur. The

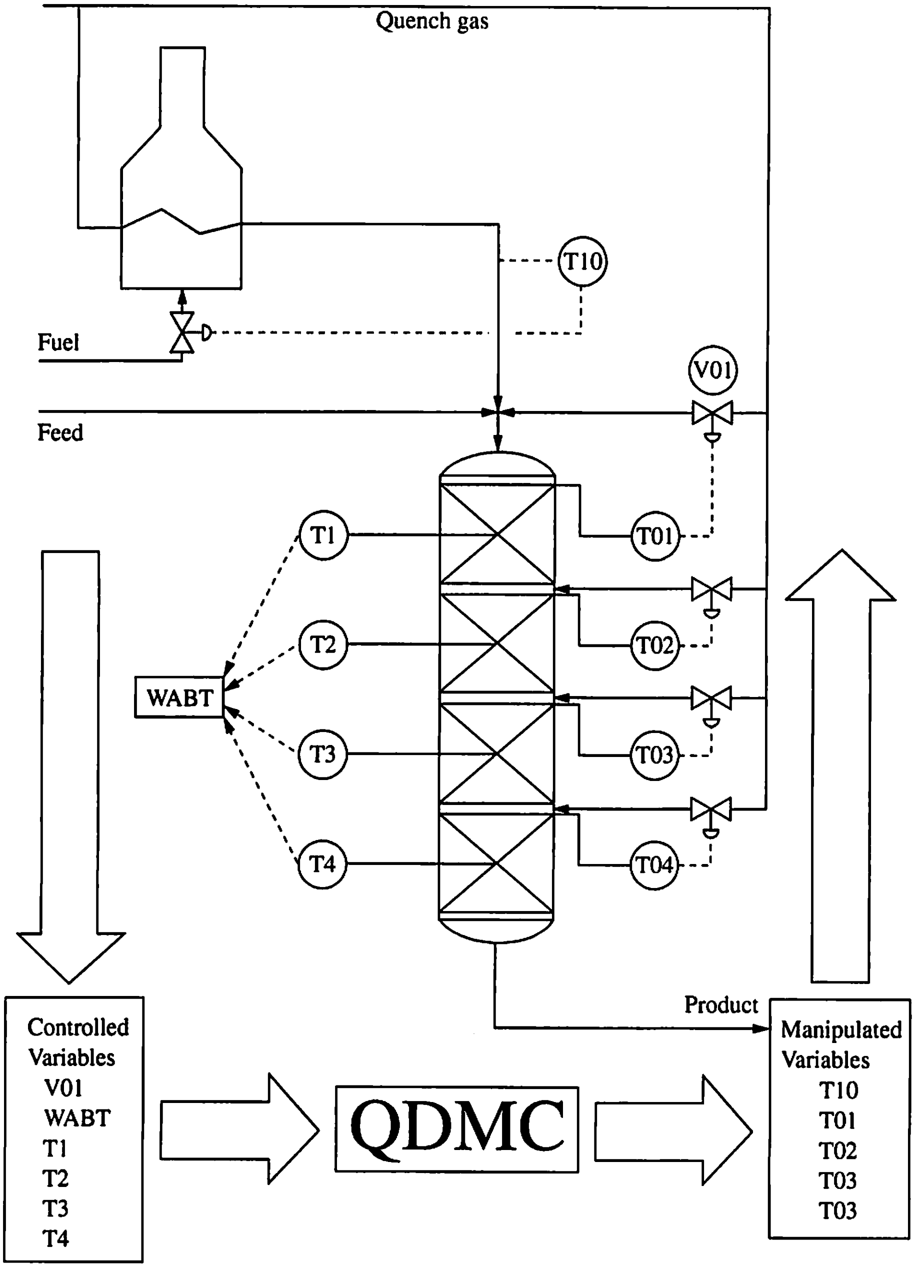


FIGURE 23.11

Hydrocracker reactor with QDMC centralized control.

reactor bed inlet temperatures are regulated by single-loop controllers that adjust the mixing of hot and cold feeds for the first reactor and adjust the injection of cold hydrogen in the second to fourth reactors. The control objectives are summarized in the following.

1. **Prevent high temperatures in each bed** Therefore, each T_i should remain below T_{max}. If this limit is closely approached or exceeded, extreme corrective action must be taken by decreasing the bed inlet temperature set point, even if

the product quality is severely upset. Note that feedback action is required for only a negative error ($T_{max} - T < 0$); this "one-sided" feedback is possible with QDMC, but not with DMC.

2. **Control total conversion** Unfortunately, the conversion of feed to products cannot be measured because of the large number of components. Even if conversion could be measured, the hundreds to thousands of reactions could not all be controlled independently. Therefore, the concept of partial control is employed (see Chapter 24), and a dominant variable is selected. For hydrocracking, the weighted average bed temperature, WABT, is often used as an inference for conversion; it weights each bed temperature by the mass of catalyst in the bed.
3. **Reducing energy consumption** An indication of this objective is the amount of material that bypasses the fired heater, because mixing cold and hot streams is less efficient than heating the total feed to the required temperature.
4. **Maintain yield and catalyst activity** Notice that many different operations (i.e., values of T_1 to T_4) could yield the same WABT; therefore, the distribution of individual bed temperatures is selected to provide the desired selectivity and catalyst life in the four reactors.
5. **Manipulated variable bounds** Every manipulated variable (set point of secondary cascade controllers) must remain within specified maximum and minimum values.

The MPC design for this process and objectives is also shown in Figure 23.11 (Kelly et al., 1988). The controlled and manipulated variables are summarized in the following table.

Set point	Manipulated variables	Controlled variables (in order of decreasing importance)
T10	Fired heater effluent	Bed 1 to 4 temperature < T_{max}
T01	Bed 1 inlet temperature	WABT deviation from set point
T02	Bed 2 inlet temperature	V01 % open deviation from set point
T03	Bed 3 inlet temperature	Bed 1 to 4 temperature distribution
T04	Bed 4 inlet temperature	

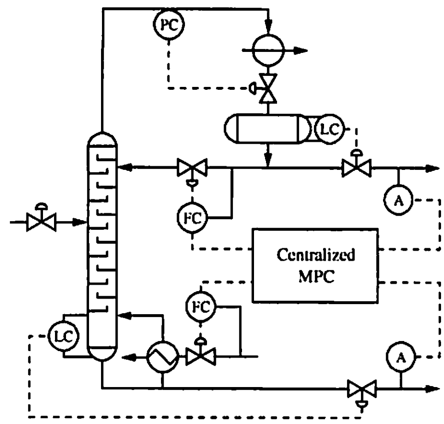
The form of the MPC used industrially by Kelly et al. (1988) was the quadratic dynamic matrix control (QDMC) with constraint handling capability. Evaluation of the dynamic performance of the design indicated that it performed very well. During evaluation tests, no bed temperature exceeded its maximum limit; the most important variable (WABT) was controlled close to its set point; the bypass valve was maintained near the desired percent open; and each of the individual bed temperatures varied about their set points (Kelly et al., 1988; Stanfelj, 1990).

EXAMPLE 23.8.

DMC control was applied to a distillation tower in Example 23.6 for situations in which no constraints were encountered. Here, QDMC is applied to the same distillation tower for situations with constraints. Again, the tower is described in Example 23.1 and modelled in equation 23.5. The set point response is considered in this example, so that these results can be compared with the unconstrained results obtained in Figure 23.9*a*.

The solution is developed with the QDMC controller described in equation (23.31) using the same values for the following parameters as used in the unconstrained case in Example 23.6.

$$\Delta t = 1 \quad \text{MM} = 5 \quad \text{NN} = 20 \quad \text{LL} = 100 \quad \text{ww}_1 = \text{ww}_2 = 1 \quad \text{qq}_1 = \text{qq}_2 = 0.02$$



Input constraint. In this situation, the reboiler duty is limited because of a maximum possible heating medium flow rate. The maximum amount of reboiled vapor is 14.1 kmol/min. The results are given in Figure 23.12*a* for a set point change in the X_D controlled variable. Because one of the manipulated variables encounters a constraint, both controlled variables cannot be maintained at their set points. Since the QDMC objective [equation (23.29)] considers both controlled variables, the controller adjusts the one remaining, unconstrained manipulated variable to minimize the sum of the (squared) errors for the distillate and bottoms compositions. Neither controlled variable achieves its set point, but each is maintained "close" to its set point. Modifications can be made to QDMC to select a priority ranking for controlled variables so that the more important can be returned to their set points when all controller variables cannot be returned to their set points (Swartz, 1995).

Output constraint. Again, the set point response for a change in the X_D set is considered. In this situation, the light key in the bottoms should be maintained below a specified limit or costly economic penalties would occur. The maximum value for X_B is 0.0205, and this limitation is included in the QDMC controller through a very severe penalty on any X_B values that exceed the limit. The results are given in Figure 23.12*b*. To reduce the disturbance to X_B due to interactions, the controller has slowed the adjustment to the manipulated variables slightly. Therefore, slightly more time is required to change the distillate composition, X_D. However, the controller achieves the dual goals of reasonably fast X_D response while X_B is maintained within its specified upper limit. This excellent performance is due to the capability of the QDMC controller and the perfect model used in this simulation example. Such excellent performance would not be expected for a realistic nonlinear process with dynamics changing due to alterations in operating conditions, but quite good performance can be achieved using centralized model predictive control.

Given the success of centralized control, the reader may wonder about using this technology for centralized control of large plants having hundreds to thousands of variables. Although theoretically possible, such large MPC controllers are not now used because of (1) the difficulty in building the models, (2) the computation time for solving the optimization problem, and (3) the challenge to the plant personnel in understanding the controller results. Typically, centralized MPC is applied to blocks of variables that have substantial interaction among themselves and weak effects on the remainder of the plant. Thus, plants can have multiple centralized, multivariable MPC and many single-loop controllers. Also single-loop controllers remain as lower-level, secondary controllers whose set points are adjusted by the higher-level MPC controllers. For example, PID controllers remain in the

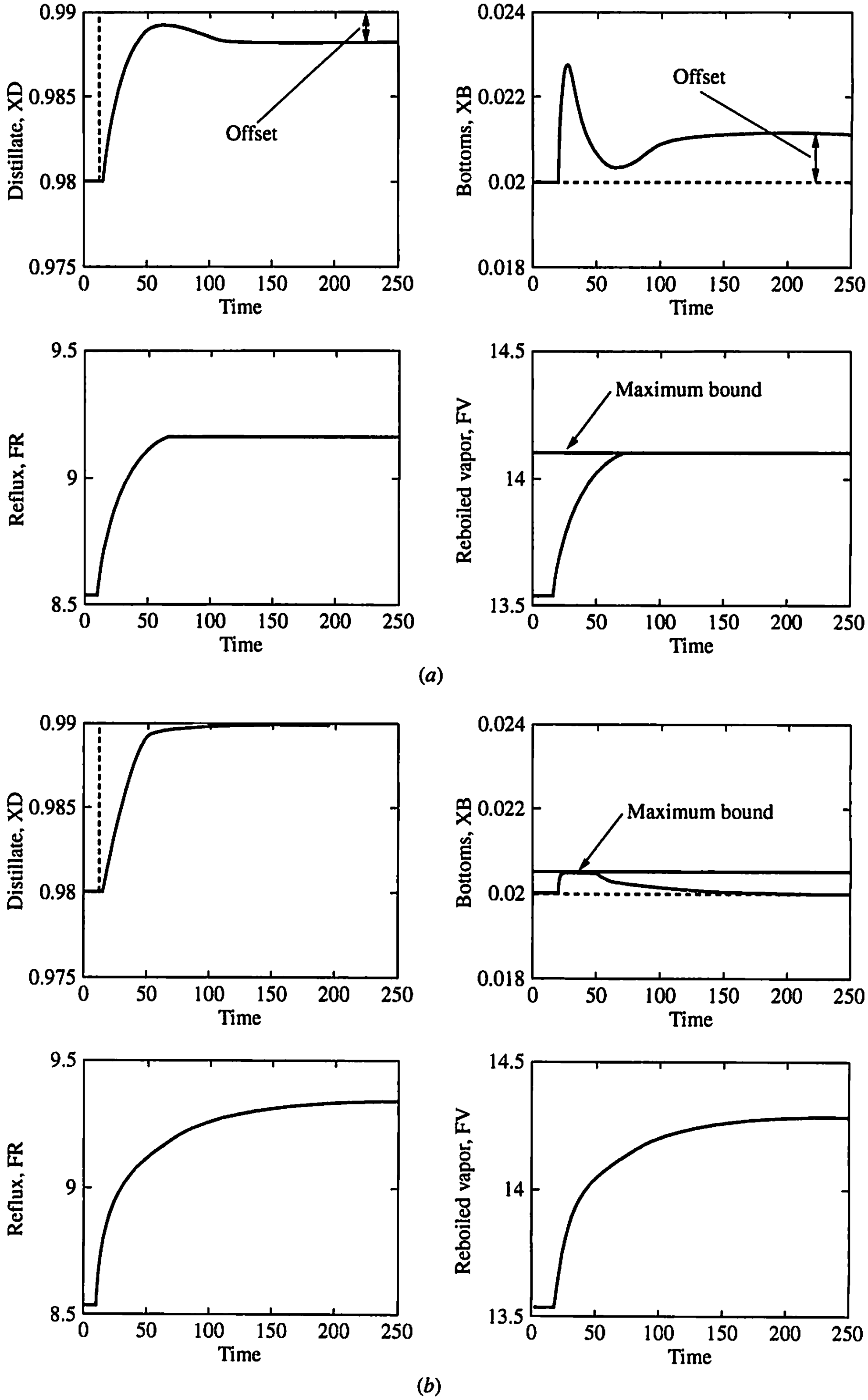


FIGURE 23.12

(*a*) Dynamic responses for closed-loop QDMC control with a maximum limit on the *manipulated* reboiled vapor rate. (Compare with Figure 23.9*a*.) (*b*) Dynamic responses for closed-loop QDMC control with a maximum limit on the *controlled* X_B. (Compare with Figure 23.9*a*.)

hydrocracker to control the furnace outlet and reactor inlet temperatures. Thus, even in the age of block centralized MPC, knowledge of single-loop control is important to the engineer!

Non-Self-Regulating Processes

The step weight model described in Section 23.3 is limited to processes that are stable and self-regulating so that they attain a steady state after a step input. As discussed in Chapter 18, many inventory processes (levels) are not self-regulatory, because they are pure integrators. The step response modelling method has been extended to integrators, and details are provided by Cutler (1982).

23.8 ◻ CONCLUSIONS

A practical method for centralized process control has been presented in this chapter. The general model predictive structure provides the framework for the control method, but the analytical design approach proves a limit to direct extension of the methods from Chapter 19. The novel modelling and numerical calculations of the dynamic matrix controller algorithm result in a method that can be applied to a wide range of processes. The addition of feedback and tuning parameters provides the basic centralized controller algorithm, with extensions possible for special situations. The performance of the dynamic matrix controller has been demonstrated to be good for single- and multivariable systems.

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ADDITIONAL RESOURCES

Centralized multivariable controllers make use of all elements in the feedback model, which can make these controllers sensitive to certain types of model mismatch. The causes of the sensitivity and experimental designs to improve model accuracy are discussed in

Kwong, C. W., and J. MacGregor, "Identification for Robust Multivariable Control: The Design of Experiments," *Automatica, 30,* 1541–1554 (1994).
Skogestad, S., and M. Morari, "Implications of Large RGA Elements on Control Performance," *IEC Res., 26,* 2323–2330 (1987).

A review of model predictive control that discusses dynamic matrix control within this structure is in

Garcia, C., D. Prett, and M. Morari, "Model Predictive Control: Theory and Practice—a Survey," *Automatica, 25,* 335–348 (1989).

An alternative approach using linear programming is reported in

Brosilow, C., and G. Zhao, "A Linear Programming Approach to Constrained Multivariable Process Control," in C. Leondes (ed.), *Control and Dynamic Systems, 27,* 141 (1986).

The full power of centralized multivariable control becomes apparent through studies of closed-loop systems. These questions build understanding of the assumptions, theory, and preliminary calculations that can be performed without preparing a complete design and simulation package.

23.1. Determine step response models (i.e., the step weights) for the following systems based on the continuous models already developed. Select appropriate values for the sample period, the input horizon, and the output horizon.
Single-variable:
(*a*) The three-tank mixing process, first-order-with-dead-time approximation (Example 6.4, base case)
(*b*) The series chemical reactors in Examples 3.3 and 4.12.
Two-variable:
(*c*) The blending process in Examples 20.6 and 20.10
(*d*) The two processes with simple and complex interactive dynamics, B1 and B2, in Example 21.4
(*e*) The distillation tower under material balance regulatory control in equation (21.2)

23.2. Calculate the dynamic matrix controller $\mathbf{K}_{\text{DMC}}$ for one of the single-loop processes already modelled in question 23.1. Select an appropriate input horizon and let ww = 1 for all controlled variables. The calculations can be performed on a spreadsheet or using a programming language. After the controller has been determined, evaluate the response of the controlled and manipulated variables to a step change in the set point without model error; this can be done by evaluating the product in equation (23.22), $[\Delta\text{MV}] = [\mathbf{K}_{\text{DMC}}][\mathbf{E}^f]$, where $[\mathbf{E}^f] = [\Delta\text{SP}]$. Begin with qq = 0, and increase it. Select an appropriate initial value for qq.

23.3. The step response model can be determined from empirical data.
(*a*) Discuss the advantages and disadvantages for using sampled values of the original data for the model.
(*b*) Discuss the procedure required and likely results of fitting the coefficients a_j in the following model to experimental data using linear least squares. Recall that this model will have between 20 and 50 coefficients.

$$Y_{k+1} = \sum_{j=1}^{k+1} a_j \Delta X_{k-j+1} + Y_0$$

(*c*) Are the dynamics of the sensor and the final element included in the models used in the design of the DMC controller?

23.4. The DMC objective function selected to be minimized is the ISE over the output horizon.
(*a*) What is the advantage of using the ISE rather than the IAE or $(\text{error})^4$?
(*b*) From a necessary condition for a minimum (the gradient is zero), derive the equation for the DMC controller in equation (23.21).

23.5. Derive the analytical model predictive controller for the following processes. For each, state whether the controller can be easily factored, and if so, select an IMC filter structure and time constant value(s) to give good dynamic performance.

(*a*) The systems B1 and B2 in Example 21.4.
(*b*) The blending process in Examples 20.6 and 20.10.
(*c*) The distillation tower with material balance regulatory control in equation (21.2).

23.6. Discuss the effect on the closed-loop performance of the following changes.
(*a*) Multiply every ww and qq by a positive constant.
(*b*) Add a constant to the DMC objective function.
(*c*) Change the units of one controlled variable, for example, the bottoms composition in Example 23.5, from mole fraction to mole percent.
(*d*) Increase all qq by the same positive factor, maintaining all ww constant.

23.7. Develop the appropriate step response model for a pure integrating level process. Describe how this could be used to model the process over a long time, without involving a summation of infinite length.

23.8. Determine all calculations for adding feedforward control for a measured disturbance to the single-loop DMC control system in Example 23.5. The answer should include a block diagram, summary of controller execution calculations, and any new models and/or modifications to the controller $\mathbf{K}_{\text{DMC}}$. The model for the disturbance is $G_d(s) = 1.0e^{-2.5s}/(5s+1)$. Also, design a feedforward controller using methods in Chapter 15 and discuss the expected difference in performance.

23.9. Determine all calculations for adding feedforward control for a measured disturbance in the feed composition to the multivariable DMC control system in Example 23.6. The answer should include a block diagram, summary of controller execution calculations, and any new models or modifications to the controller, $\mathbf{K}_{\text{DMC}}$. The model for the disturbance is given in equation (23.5). Also, design a feedforward controller using methods in Chapter 15, and discuss the expected difference in performance.

23.10. Criteria for zero steady-state offset from set point are presented in Chapter 19 for IMC and Smith predictor designs. Determine the criteria for the DMC system to achieve zero steady-state offset for a steplike disturbance.

23.11. Suppose that slower set point response was desired, but fast disturbance response was required. How could you modify the DMC control system design to accommodate this performance requirement? (Hint: Review Chapter 19 for an approach to achieve this performance.)

23.12. The DMC controller was described in this chapter using step response models to calculate the model to compare with the feedback measurement and to calculate the future performance without control, CV_i^f.
(*a*) Describe how the discrete models derived in Appendix F could be used for these calculations.
(*b*) Could these models also be used to determine $\mathbf{K}_{\text{DMC}}$?